NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)
Pampady, Thiruvilwamala, Thrissur Dist, Kerala-680588

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIAL

ME 402 DESIGN OF MACHINE ELEMENTS-II

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- Established in: 2002
- Course offered : B.Tech in Mechanical Engineering
- Approved by AICTE New Delhi and Accredited by NAAC

Affiliated to the University of Dr. A P J Abdul Kalam Technological University

DEPARTMENT VISION

Producing internationally competitive Mechanical Engineers with social responsibility & sustainable employability through viable strategies as well as competent exposure oriented quality education.

DEPARTMENT MISSION

1. Imparting high impact education by providing conductive teaching learning environment.
2. Fostering effective modes of continuous learning process with moral & ethical values.
3. Enhancing leadership qualities with social commitment, professional attitude, unity, team spirit & communication skill.
4. Introducing the present scenario in research & development through collaborative efforts blended with industry & institution.

PROGRAMME EDUCATIONAL OBJECTIVES

PEO1: Graduates shall have strong practical & technical exposures in the field of Mechanical Engineering & will contribute to the society through innovation & enterprise.

PEO2: Graduates will have the demonstrated ability to analyze, formulate & solve design engineering / thermal engineering / materials & manufacturing / design issues & real life problems.

PEO3: Graduates will be capable of pursuing Mechanical Engineering profession with good communication skills, leadership qualities, team spirit & communication skills.

PEO4: Graduates will sustain an appetite for continuous learning by pursuing higher education & research in the allied areas of technology.

PROGRAM OUTCOMES (POS)

Engineering Graduates will be able to:

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation

of data, and synthesis of the information to provide valid conclusions.

5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO)

PSO1: graduates able to apply principles of engineering, basic sciences & analytics including multi variant calculus & higher order partial differential equations..

PSO2: Graduates able to perform modeling, analyzing, designing & simulating physical systems, components & processes.

PSO3: Graduates able to work professionally on mechanical systems, thermal systems & production systems.

COURSE OUTCOMES

CO1	The students will be able to acquire knowledge and design of different types of clutches and brakes
CO2	The students will be able to understand the basics of bearings, types of bearing, lubrication system, design considerations and design of bearings.
CO3	The students will be able to understand the concept of spur gear and the basic procedure in design of spur gear.

CO4	The students will be able to understand the concept of helical gear, bevel gear, worm and worm wheel & the basic procedure in design of above said gears.
CO5	The students will be able to acquire knowledge and design of flat belt, v belt and chains.
CO6	The students will be able to acquire basic knowledge and design of Connecting rod and Pressure vessels.

MAPPING OF COURSE OUTCOMES WITH PROGRAM OUTCOMES

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	2	2	3	-	2	-	-	-	-	-	-	2	2	3	2
CO2	2	2	3	-	2	-	-	-	-	-	-	2	2	3	2
CO3	2	2	3	-	2	-	-	-	-	-	-	2	2	3	2
CO4	2	2	3	-	2	-	-	-	-	-	-	2	2	3	2
CO5	2	2	3	-	2	-	-	-	-	-	-	2	2	3	2
CO6	2	2	3	-	2	-	-	-	-	-	-	2	2	3	2

Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

SYLLABUS

Course No.	Course Name	L-T-P-Credits	Year of Introduction
ME 402	Design of Machine Elements-II	3-0-0-3	2016

Prerequisite: ME401 Design of Machine Elements-I

Course Objectives:

- To provide basic design methods for clutches, brakes, belt drives, bearings, gears and connecting rod.
- To introduce the design modifications to be considered for ease of manufacturing.

Syllabus

Design of single plate clutches, multiple disc clutches, cone clutch, centrifugal clutch, block brake, band brake, band and block brake, internal expanding shoe brake, rolling contact bearing, sliding contact bearing, spur gear, helical gear , bevel gear, worm and worm wheel, design of flat belt, design of V-belt drives, selection of roller chains, connecting road, design recommendations for forgings, castings, welded products, rolled sections, turned parts, screw machined products, parts produced on milling machines.

Expected outcome:

The students will be able to

1. Apply design procedures for industrial requirements.
2. Design machine components to ease the manufacturing limitations.

Text Books:

1. J. E. Shigley, Mechanical Engineering Design, McGraw Hill,2003
2. Jalaludeen , Machine Dsign, Anuradha Publications, 2016
3. V.B.Bhandari, Design of Machine elements, McGraw Hill, 2016

References Books:

1. Juvinall R.C & Marshek K.M., Fundamentals of Machine Component Design, John Wiley, 2011
2. M. F. Spotts, T. E. Shoup, Design of Machine Elements, Pearson Education, 2006
3. Rajendra Karwa, Machine Design , Laxmi Publications (P) LTD, New Delhi, 2006
4. Siegel, Maleev& Hartman, Mechanical Design of Machines, International Book Company, 1983

Data books permitted for reference in the examination:

1. K. Mahadevan, K.Balaveera Reddy, Design Data Hand Book, CBS Publishers & Distributors, 2013
2. Narayana Iyengar B.R & Lingaiah K, Machine Design Data Handbook, Tata McGraw Hill, 1984
3. PSG Design Data, DPV Printers, Coimbatore, 2012

Module	Contents	Hours	End Sem. Exam Marks
I	Clutches – friction clutches, design considerations, multiple disc clutches, cone clutch, centrifugal clutch	2	15%
	Brakes- Block brake, band brake, band and block brake, internal expanding shoe brake	3	
II	Rolling contact bearing- Design of bearings, Types, Selection of a bearing type, bearing life, static and dynamic load capacity, axial and radial loads, selection of bearings, dynamic equivalent load	4	15%
	Sliding contact bearing- lubrication, lubricants, viscosity, Journal bearings, hydrodynamic theory, Sommerfield number, design considerations, heat balance, bearing housing and mountings	4	
	FIRST INTERNAL EXAM		
III	Gears- classification, Gear nomenclature, Tooth profiles, Materials of gears, Law of gearing (review only), virtual or formative number of teeth, gear tooth failures, Beam strength, Lewis equation, Buckingham's equation for dynamic load, wear load, endurance strength of tooth, surface durability, heat dissipation – lubrication of gears – Merits and demerits of each type of gears.	3	15%
	Design of spur gear	3	
IV	Design of helical gear	2	15%
	Design of bevel gear	2	
	Design of worm & worm wheel	3	
	SECOND INTERNAL EXAM		
V	Design of flat belt- materials for belts, slip of the belts, creep, centrifugal tension	3	20%
	Design of V-belt drives, Advantages and limitations of V-belt drive	3	
	Selection of roller chains, power rating of roller chains, galling of roller chains, polygonal action, silent chain.	3	
VI	Connecting rod – material, connecting rod shank, small end, big end, connecting rod bolts, inertia bending stress, piston	5	20%
	Pressure vessels, thin cylinders, Thick cylinder equation, open and closed cylinders	2	

QUESTION PAPER PATTERN

Note : Use of approved data book is permitted

Maximum marks: 100 **Time: 3 hrs**

The question paper should consist of three parts

Part A
There should be 3 questions from module I and II and at least 1 question from each module
Each question carries 15 marks
Students will have to answer any 2 questions out of 3 (2X15 marks =30 marks)

Part B
There should be 3 questions from module III and IV and at least 1 question from each module
Each question carries 15 marks
Students will have to answer any 2 questions out of 3 (2X15 marks =30 marks)

Part C
There should be 3 questions from module V and VI and at least 1 question from each module
Each question carries 20 marks
Students will have to answer any 2 questions out of 3 (2X20 marks =40 marks)

Note: Each question can have a maximum of four sub questions, if needed.

		SET 1	Total Pages: 2

Reg No.:_______________		Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

EIGHTH SEMESTER B.TECH DEGREE EXAMINATION, MAY2020

Course Code: ME 402

Course Name: Design of Machine Elements-II

Max. Marks: 100		Duration:2 Hour 15 Minutes

Use of design data book is permitted

Missing data may be suitably assumed

PART A

		Answer any two full questions, each carries 15 mark	Marks
1	a)	A multiple disc plate consists of five steel plates and four bronze plates. The inner and outer diameter of the friction discs are 76 mm and 150 mm respectively. The co-efficient of friction is 0.1 and the intensity of pressure on friction lining is limited to 0.3 N/ mm^2. Calculate the force to engage clutch and power transmitting capacity at 750 rpm.	(10)
	b)	Give an account on self-energizing and self-locking conditions in brakes.	(5)
2	a)	A single row angular contact ball bearing number 310 is used for an axial flow compressor. The bearing is to carry a radial load of 2500 N and an axial or thrust load of 1500 N. Assuming light shock load. Determine the rating life of the bearing.	(10)
	b)	Why it is necessary to dissipate the heat generated when clutches operate.	(5)
3		A journal bearing of a centrifugal pump running at 1740 rpm, has to support a steady load of 8 kN. The journal diameter from trial calculation is found to be 120 mm. Design suitable journal bearing for the pump to operate under hydrodynamic condition.	(15)

PART B

Answer any two full questions, each carries 15 marks.

4		Design a pair of spur gears to transmit 20 kW at a pinion speed of 1400 rpm. The transmission ratio is 4. The material for pinion and gear are 15Ni2Cr Mo15 and C45 respectively. The pressure angle of gear and pinion is 20°.	(15)
5	a)	Design 20 degree involute worm gear to transmit 10 kW with worm rotating at 1400 rpm and to obtain a speed reduction of 12:1. The distance between the shafts is 225 mm.	(10)
	b)	Differentiate crown gear and miter gear.	(5)
6		Design a suitable gear drive to transmit 2 kW at 1440 rpm between two shafts at right angles. Select suitable materials for the gears.	(15)
		PART C	
		Answer any two full questions, each carries 20 marks.	
7	a)	Design a connecting rod for a petrol engine for the following data: Diameter of the piston = 68 mm Stroke length = 80 mm Length of connecting rod = 160 mm Maximum explosion pressure = 3.5 N/ mm^2 Mass of reciprocating parts = 2.5 kg Speed = 4000 rpm Compression ratio = 8 :1	(16)
	b)	In what way a thin cylinder is differed from a thick cylinder.	(4)
8	a)	Select a flat belt to drive a mill at 250 rpm from a 10 KW, 730 rpm motor. Centre distance is to be around 2m. The mill shaft pulley is of 1m diameter.	(12)
	b)	Write short notes on galling of roller chains.	(4)
	c)	Discuss the effect of centrifugal tension on power transmission in belt drive.	(4)
9	a)	Design a chain drive to actuate a compressor from a 10KW electric motor at 960 rpm. The compressor speed is to be 350 rpm. Minimum centre distance should be	(12)

		0.5m. Motor is mounted on an auxiliary bed. Compressor is to work for 8 hours/day.	
	b)	A steel tank having an internal diameter of 2.5 m is required to withstand an internal fluid pressure of 25N/mm^2. The tensile and compressive elastic strength of the material is 200N/mm^2. Find the thickness of the wall of the tank if the working value of the circumferential stress is two-third of the tensile elastic strength.	(8)
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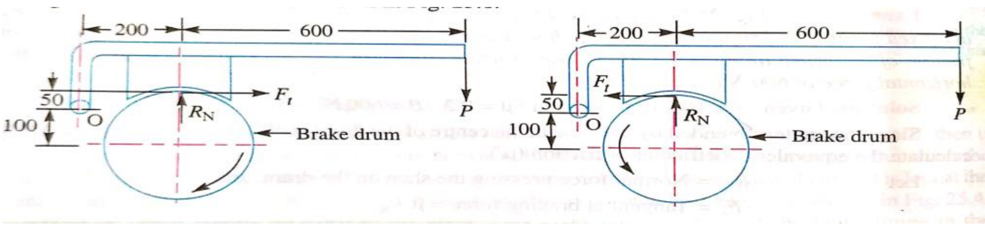
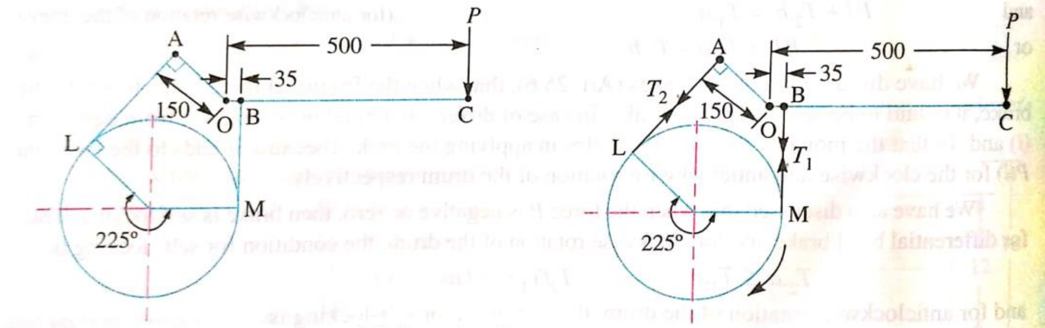
QUESTION BANK

Knowledge Level (KL)	K1 : Remembering	K3:Applying	K5: Evaluating
Course Outcome (CO)	K2: Understanding	K4: Analysing	K6: Creating

MODULE I			
Q:NO:	**QUESTIONS**	**CO**	**KL**
1	What are the general considerations that must be observed in the design of friction clutches?	CO1	K2
2	A single plate clutch, effective on both sides, is required to transmit 25 kW at 4000 r.p.m. Determine the inner and outer diameter of friction surface if the co-efficient of friction is 0.23, ratio of diameter is 1.25 and the maximum pressure is not to exceed o.1 N/mm2. Also, determine the axial thrust to be provided by springs. Assume the theory of uniform wear.	CO1	K5
3	The block brake, as shown in Figure, provides a braking torque of 360 N-m. The diameter of the brake drum is 300 mm. The co-efficient of friction is 0.3. Find: (i) The force (P) to be applied on the end of the lever for the clockwise and counter clockwise rotation of the brake drum; and (ii) The location of the pivot or fulcrum to make the brake self-locking for the clockwise rotation of the brake drum. 	CO1	K5
4	Give an account on self-energizing and self-locking conditions in brakes.	CO1	K2
5	How does the function of a brake differ from that of a clutch?	CO1	K2
6	A multiple disc plate consists of five steel plates and four bronze plates. The inner and outer diameter of the friction discs are 76mm and 150mm respectively. The co-efficient of friction is 0.1 and the intensity of pressure on friction lining is limited to 0.3 N/ mm^2. Calculate the force to engage clutch and power transmitting capacity at 750 rpm	CO1	K5
7	A differential band brake, as shown in fig, has an angle of contact of 225^0. The band has a compressed woven lining and bears against a cast iron drum of 350 mm diameter. The brake is to sustain a torque of 350 N-m and the coefficient of friction between the band and the drum is 0.3. Find 1. The necessary fore (P) for the clockwise and anti-clockwise rotation of the drum; and 2. The value of 'OA' for the brake to be self-locking, when the drum rotates clockwise.	CO1	K6

	 All dimensions in mm.		
8	Why it is necessary to dissipate the heat generated when clutches operate.	CO1	K2
9	What are the procedures involved in designing a block brake	CO1	K2
10	A centrifugal clutch is to be designed 15KW at 900 rpm. The shoes are 4 in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley is 150 mm. The shoes are lined with ferrodo for which the co-efficient of friction may be taken as 0.23. Determine (1) mass of shoes (2) size of the shoes	CO1	K6
11	In a band and block brake, the band is lined with 12 blocks, each of which subtends an angle of 20^0 at the drum centre. The two ends of the band are attached to a differential brake-lever at the distance of 40 mm and 160 mm from fulcrum in such a way that the longer distance is nearer to the actuating force. Find the force required at the end of the lever 1.2 metre long from fulcrum to give a torque of 4kN-m. The diameter of the brake drum is 1 metre and the coefficient of friction between the blocks and the drum is 0.25.	CO1	K6
12	Give an account on self-energizing and self-locking conditions in brakes	CO1	K2
	MODULE II		
1	A journal bearing of a centrifugal pump running at 1740 rpm, has to support a steady load of 8kN. The journal diameter from trial calculation is found to be 120 mm. Design suitable journal bearing for the pump to operate under hydrodynamic condition	CO2	K6
2	What is meant by the rating life of a bearing?	CO2	K2
3	A bearing is required to carry 4500N stationary radial load. The shaft rotates at 1000 rpm and the life desired is 30000 hours. The running conditions are steady without shock loading. Select a suitable bearing	CO2	K6
4	Describe about Sommerfield number.	CO2	K2
5	A single row angular contact ball bearing number 310 is used for an axial flow compressor. The bearing is to carry a radial load of 2500 N and an axial or Thrust load of 1500 N. Assuming light shock load. Determine the rating life of the bearing.	CO2	K6
6	Give an account on dynamic equivalent load in bearings	CO2	K2
7	A bearing is to carry 7,000 N radial load and 4,000 N thrust load. The shaft rotates at 1500 rpm. For smooth load and 8 hrs per day service for 5 years (a) Select a deep groove ball bearing; (b) Compute the 90% of the life bearing and the probability of the selected bearing serving the above life	CO2	K6

8	Explain the principle hydrodynamic lubrication	CO2	K2
9	Design a journal bearing for a centrifugal pump to the following specifications; diameter of the journal = 75 mm, speed of the journal = 1140 rpm and load n each journal=11500N	CO2	K5
10	Define the term bearing characteristic number	CO2	K2
MODULE III			
1	State and prove law of gearing	CO3	K2
2	Why gear design is to be checked for beam strength and compressive strength?	CO3	K3
3	Design a pair of spur gears to transmit 20 kW at a pinion speed of 1400 rpm. The transmission ratio is 4. The material for pinion and gear are 15Ni2Cr Mo15 and C45 respectively. The pressure angle of gear and pinion is 20°.	CO3	K5
4	Discuss the modes of gear failures	CO3	K2
5	Explain Lewis equation	CO3	K2
6	What condition must be satisfied for a pair of spur gears to have a constant velocity ratio?	CO3	K2
7	Design the teeth for a pair of cut-teeth spur gears with 20° full depth teeth to be made of nickeled chromium steel. The gears are to be transmitting 45 kW at 3000 rpm of the pinion. The pinion has 21 teeth and the gear has 25 teeth. Find the module, face width and diameter for continuous service	CO3	K5
8	What do you mean by backlash in gears?	CO3	K2
9	Write short notes on Buckingham's equation for dynamic load	CO3	K2
10	What is dynamic load in gear drive? How it could be avoided?	CO3	K2
11	Design the teeth for a pair of cut-teeth spur gears with 20° full depth teeth to be made of nickeled chromium steel. The gears are to be transmitting 45 kW at 3000 rpm of the pinion. The pinion has 21 teeth and the gear has 25 teeth. Find the module, face width and diameter for continuous service.	CO3	K6
12	Discuss the modes of gear failures	CO3	K2
MODULE IV			
1	Differentiate crown gear and miter gear	CO4	K2
2	What is virtual number of teeth in bevel gear and state its significance?	CO4	K2
3	A pair of parallel helical gears consists of an 18 teeth pinion meshing with a 45 teeth gear and 75 kW power at 2000 rpm is supplied to the pinion. The normal module is 6mm and pressure angle 20°. Determine the tangential, radial and axial components of the resultant tooth force between the meshing teeth, if the helix angle is 23°.	CO4	K5

4	Design 20 degree involute worm gear to transmit 10 kW with worm rotating at 1400 rpm and to obtain a speed reduction of 12:1. The distance between the shafts is 225 mm	CO4	K6
5	Give a brief account for skew gears	CO4	K2
6	What is helix angle? How this angle differentiate helical gear from spur gear.	CO4	K2
7	A pair of helical gears with 30° helix angle is used to transmit 15 kw at 10,000 rpm of the pinion. The velocity ratio is 4:1. Both the gears are to be made of hardened steel of static strength 100 N/mm^2. The gears are 20° stub and the pinion is to have 24 teeth. The face width may be taken as 14 times the module. Find the module and face width from the stand point of strength and check the gears for wear.	CO4	K5
8	A pair of straight bevel gear is required to transmit 10 kW at 500 rpm from the motor shaft to another shaft at 250 rpm. The pinion has 24 teeth. The pressure angle is 20 degree. If the shaft axes are at right angles to each other, find the module, face width, addendum outside diameter and slant height. The gears are capable of withstanding a static stress of 60 MPa. The face width may be taken as ¼ of the slant height of the pitch cone .	CO4	K5
9	1 kW of power is transmitted to the worm shaft at 720 rpm. The number of threads of worm is four with 50 mm pitch circle diameter. The worm wheel has 30 teeth with 5 mm module. Calculate the rubbing velocity, Efficiency of the worm gear and the power lost in friction.	CO4	K5
	MODULE V		
1	Discuss the effect of centrifugal tension on power transmission in belt drive.	CO5	K4
2	When do we prefer a V-belt to a flat-belt?	CO5	K2
3	Design a flat belt drive for a stone crushing machine. The power is transmitted from a 20 kW motor running at 1200 rpm to the machine running at 260 rpm. The diameter of machine pulley is 1 m.	CO5	K6
4	Select a suitable V-belt and design the drive for a wet grinder. Power is available from a 0.5KW motor running at 750 rpm. Drum speed is to be about 100 rpm. Drive is to be compact.	CO5	K5
5	Explain the term “Crowning of pulley” and also state” law of belting”.	CO5	K2
6	Analyze the chordal or polygonal action of a roller chain	CO5	K4
7	Select a flat belt to drive a mill at 250 rpm from a 10 KW, 730 rpm motor. Centre distance is to be around 2m. The mill shaft pulley is of 1m diameter.	CO5	K5
8	A motor driven blower is to run at 650 rpm driven by an electric motor of 7.5KW at 1800 rpm. Design a V-belt.	CO5	K5
9	Explain briefly about creep in belts and ply in belts.	CO5	K2
10	Write short notes on galling of roller chains.	CO5	K2
	MODULE VI		

1	Explain why I section is usually preferred in the case of a connecting rod?	CO6	K2
2	Under what force, the big end bolts and caps are designed.	CO6	K2
3	Design a connecting rod of I cross section for an IC engine running at 1800 rpm and developing a max pressure of 3.15 N/mm^2. The diameter of the piston is 100 mm, mass of the reciprocating parts per cylinder is 2.25 Kg, length of the connecting rod is 380mm, stroke of the piston is 190mm and compression ratio is 6:1. Take a factor of safety of 6 for the design. The max allowable bearing pressure at big end and the small end are respectively 10 N/mm^2 and 15 N/mm^2. The density of material of the rod may be taken as 8000 Kg/m^3 and allowable stress in the bolts as 85N/mm^2 and the cap as 80 N/mm^2.	CO6	K6
4	A seamless cylinder with a storage capacity of 0.025m^3 is subjected to an internal pressure of 8 MPa. The length of the cylinder is twice its internal diameter. The cylinder is made of plain carbon steel 20C8. (Ultimate stress is 380 MPa and Factor of safety is 2.5). Determine the dimensions of the cylinder.	CO6	K5
5	Discuss the materials commonly used for making the connecting rod. Give reasons.	CO6	K2
6	In what way a thin cylinder is differed from a thick cylinder.	CO6	K4
7	Design the connecting rod of a steam engine to the following data: Length of the connecting rod = 825 mm, Dia. of the crank pin= 155 mm. Dia. of the cross head pin= 95mm, Max. Load on the pin= 148720N. The rod is to be made of circular cross section and made hollow by boring a central hole of 28mm Dia. throughout the length.	CO6	K6
8	A steel tank having an internal diameter of 2.5 m is required to withstand an internal fluid pressure of 25N/mm^2. The tensile and compressive elastic strength of the material is 200N/mm^2. Find the thickness of the wall of the tank if the working value of the circumferential stress is two-third of the tensile elastic strength	CO6	K5
9	Determine the dimensions of cross-section of the connecting rod for a diesel engine with the following data; Cylinder bore= 100mm, length of connecting rod = 350mm, Max gas pressure = 4 MPa and take factor of safety as 6.	CO6	K5
10	Determine the dimensions of small end and big end bearings of the connecting rod for a diesel engine with the following data; Cylinder bore=100mm, Max gas pressure = 4 MPa, (l/d) ratio for piston pin bearing=2, (l/d) ratio for crank pin bearing=1.3, Allowable bearing pressure for piston pin bearing=12 MPa, Allowable bearing pressure for crank pin bearing = 7.5 MPa	CO6	K6
11	The following data is given for the cap and bolts of the big end of connecting rod; Engine speed = 1800 rpm, Length of connecting rod=350 mm, Length of stroke=175 mm, Mass of reciprocating parts =2.5 kg, length of crank pin=76mm, diameter of crank pin=58mm, thickness of bearing bush=3mm, Permissible tensile stress for bolt= 60 MPa and Permissible tensile stress for cap= 80 MPa. Calculate the nominal diameter of the bolt s and thickness of cap for the big end.	CO6	K6

24

CHAPTER

Clutches

24.1 Introduction

A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft. The use of a clutch is mostly found in automobiles. A little consideration will show that in order to change gears or to stop the vehicle, it is required that the driven shaft should stop, but the engine should continue to run. It is, therefore, necessary that the driven shaft should be disengaged from the driving shaft. The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever.

24.2 Types of Clutches

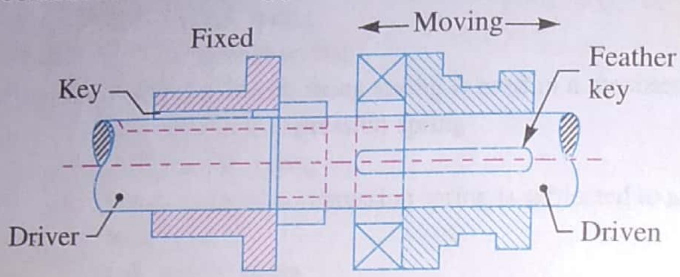
Following are the two main types of clutches commonly used in engineering practice :

1. Positive clutches, and 2. Friction clutches.

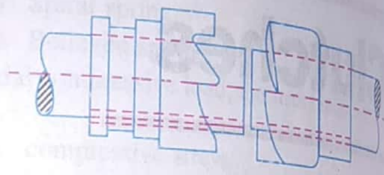
We shall now discuss these clutches in the following pages.

24.3 Positive Clutches

The positive clutches are used when a positive drive is required. The simplest type of a positive clutch is a ***jaw*** or ***claw clutch***. The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws. It consists of two halves, one of which is permanently fastened to the



(*a*) Square jaw clutch.



(*b*) Spiral jaw clutch.

Fig. 24.1. Jaw clutches.

driving shaft by a sunk key. The other half of the clutch is movable and it is free to slide axially on the driven shaft, but it is prevented from turning relatively to its shaft by means of feather key. The jaws of the clutch may be of square type as shown in Fig. 24.1 (*a*) or of spiral type as shown in Fig. 24.1 (*b*).

A square jaw type is used where engagement and disengagement in motion and under load is not necessary. This type of clutch will transmit power in either direction of rotation. The spiral jaws may be left-hand or right-hand, because power transmitted by them is in one direction only. This type of clutch is occasionally used where the clutch must be engaged and disengaged while in motion. The use of jaw clutches are frequently applied to sprocket wheels, gears and pulleys. In such a case, the non-sliding part is made integral with the hub.

24.4 Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the drive shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually bring the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that :

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.
2. The heat of friction should be rapidly *dissipated and tendency to grab should be at a minimum.
3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

24.5 Material for Friction Surfaces

The material used for lining of friction surfaces of a clutch should have the following characteristics :

* During operation of a clutch, most of the work done against frictional forces opposing the motion is liberated as heat at the interface. It has been found that at the actual point of contact, the temperature as high as 1000°C is reached for a very short duration (*i.e.* for 0.0001 second). Due to this, the temperature of the contact surfaces will increase and may destroy the clutch.

1. It should have a high and uniform coefficient of friction.
2. It should not be affected by moisture and oil.
3. It should have the ability to withstand high temperatures caused by slippage.
4. It should have high heat conductivity.
5. It should have high resistance to wear and scoring.

The materials commonly used for lining of friction surfaces and their important properties are shown in the following table.

Table 24.1. Properties of materials commonly used for lining of friction surfaces.

Material of friction surfaces	*Operating condition*	*Coefficient of friction*	*Maximum operating temperature (°C)*	*Maximum pressure (N/mm²)*
Cast iron on cast iron or steel	dry	0.15 – 0.20	250 – 300	0.25– 0.4
Cast iron on cast iron or steel	In oil	0.06	250 – 300	0.6 – 0.8
Hardened steel on Hardened steel	In oil	0.08	250	0.8 – 0.8
Bronze on cast iron or steel	In oil	0.05	150	0.4
Pressed asbestos on cast iron or steel	dry	0.3	150 – 250	0.2 – 0.3
Powder metal on cast iron or steel	dry	0.4	550	0.3
Powder metal on cast iron or steel	In oil	0.1	550	0.8

24.6 Considerations in Designing a Friction Clutch

The following considerations must be kept in mind while designing a friction clutch.

1. The suitable material forming the contact surfaces should be selected.
2. The moving parts of the clutch should have low weight in order to minimise the inertia load, especially in high speed service.
3. The clutch should not require any external force to maintain contact of the friction surfaces.
4. The provision for taking up wear of the contact surfaces must be provided.
5. The clutch should have provision for facilitating repairs.
6. The clutch should have provision for carrying away the heat generated at the contact surfaces.
7. The projecting parts of the clutch should be covered by guard.

24.7 Types of Friction Clutches

Though there are many types of friction clutches, yet the following are important from the subject point of view :

1. Disc or plate clutches (single disc or multiple disc clutch),
2. Cone clutches, and
3. Centrifugal clutches.

We shall now discuss these clutches, in detail, in the following pages.

Note : The disc and cone clutches are known as ***axial friction clutches***, while the centrifugal clutch is called ***radial friction clutch***.

24.8 Single Disc or Plate Clutch

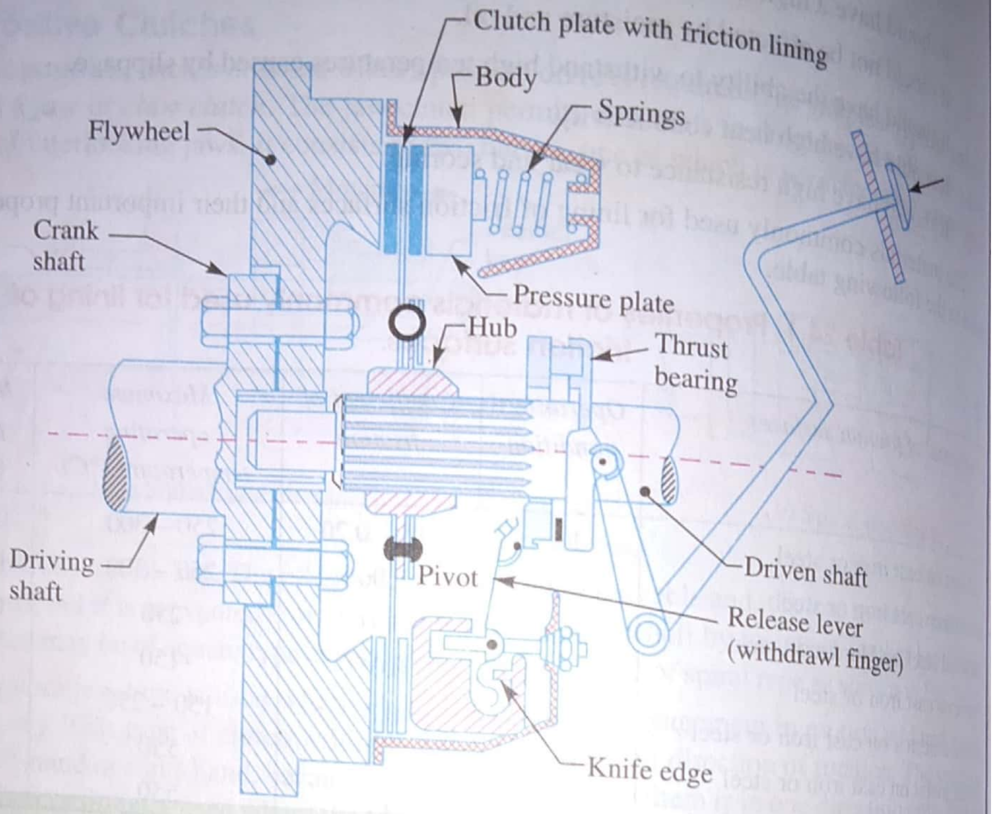


Fig. 24.2. Single disc or plate clutch.

A single disc or plate clutch, as shown in Fig 24.2, consists of a clutch plate whose both sides are faced with a frictional material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.

When a car hits an object and decelerates quickly the objects are thrown forward as they continue to move forwards due to inertia.

The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

24.9 Design of a Disc or Plate Clutch

Consider two friction surfaces maintained in contact by an axial thrust (W) as shown in Fig. 24.3 (*a*).

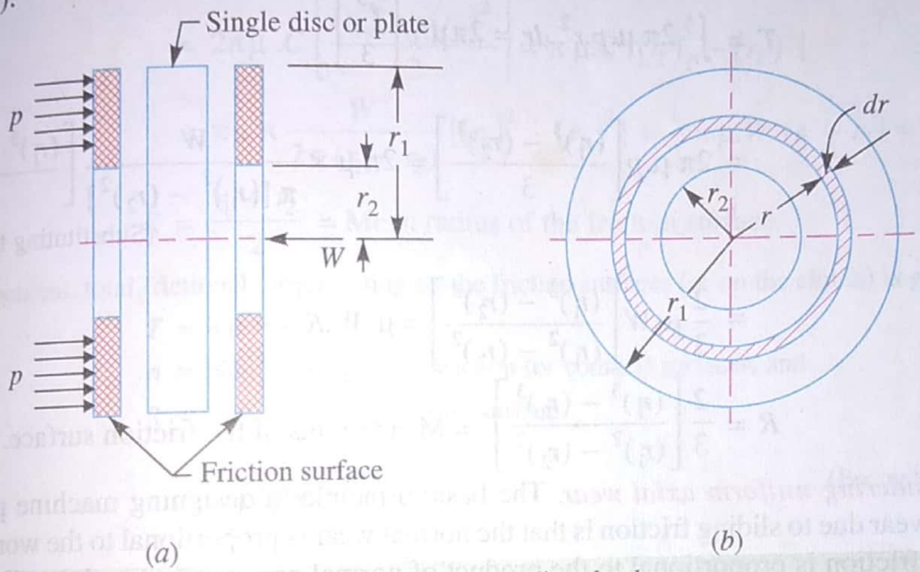


Fig. 24.3. Forces on a disc clutch.

Let
T = Torque transmitted by the clutch,
p = Intensity of axial pressure with which the contact surfaces are held together,
r_1 and r_2 = External and internal radii of friction faces,
r = Mean radius of the friction face, and
μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 24.3 (*b*).

We know that area of the contact surface or friction surface

$$= 2\pi\, r.dr$$

∴ Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu \times \delta W = \mu.p \times 2\pi\, r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\pi\, r.dr \times r = 2\,\pi\,\mu\, p.\, r^2.dr$$

We shall now consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform axial wear.

1. ***Considering uniform pressure.*** When the pressure is uniformly distributed over the entire area of the friction face as shown in Fig. 24.3 (*a*), then the intensity of pressure,

$$p = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

where $\qquad W$ = Axial thrust with which the friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\pi\ \mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total friction torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\ \mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\ \mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right] = 2\pi\ \mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

... (Substituting the value of p)

$$= \frac{2}{3}\ \mu.\ W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu\ .W.R$$

where $\qquad R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$ = Mean radius of the friction surface.

2. *Considering uniform axial wear.* The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure (p) and the sliding velocity (V). Therefore,

Normal wear ∝ Work of friction ∝ $p.V$

or $\qquad p.V = K$ (a constant) or $p = K/V \qquad$ ...(*i*)

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing-in process continues until the product $p.V$ is constant over the entire surface. After this, the wear will be uniform as shown in Fig. 24.4.

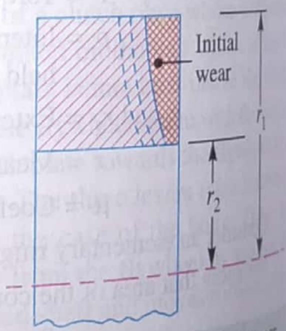


Fig. 24.4. Uniform axial wear.

Let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r = C \text{ (a constant) or } p = C/r$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total force acing on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi\ C\ dr = 2\pi\ C\ [r]_{r_2}^{r_1} = 2\pi\ C\ (r_1 - r_2)$$

or

$$C = \frac{W}{2\pi\ (r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\,\mu.p.r^2.dr = 2\pi\,\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr \qquad ...(\because p = C/r)$$

∴ Total frictional torque acting on the friction surface (or on the clutch),

$$T = \int_{r_2}^{r_1} 2\pi\,\mu\, C.r.dr = 2\pi\,\mu C\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\,.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right] = \pi\,\mu.C\,[(r_1)^2 - (r_2)^2]$$

$$= \pi\mu \times \frac{W}{2\pi\,(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of the friction surface.

Notes : 1. In general, total frictional torque acting on the friction surfaces (or on the clutch) is given by

$$T = n.\mu.W.R$$

where n = Number of pairs of friction (or contact) surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{... (For uniform wear)}$$

2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc clutch has two pairs of surfaces in contact (*i.e.* $n = 2$).

3. Since the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, therefore equation (*ii*) may be written as

$$p_{max} \times r_2 = C \quad \text{or} \quad p_{max} = C / r_2$$

4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore equation (*ii*) may be written as

$$p_{min} \times r_1 = C \quad \text{or} \quad p_{min} = C / r_1$$

5. The average pressure (p_{av}) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]}$$

6. In case of a new clutch, the intensity of pressure is approximately uniform, but in an old clutch, the uniform wear theory is more approximate.

7. The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

A twin disk clutch

24.10 Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 24.5, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

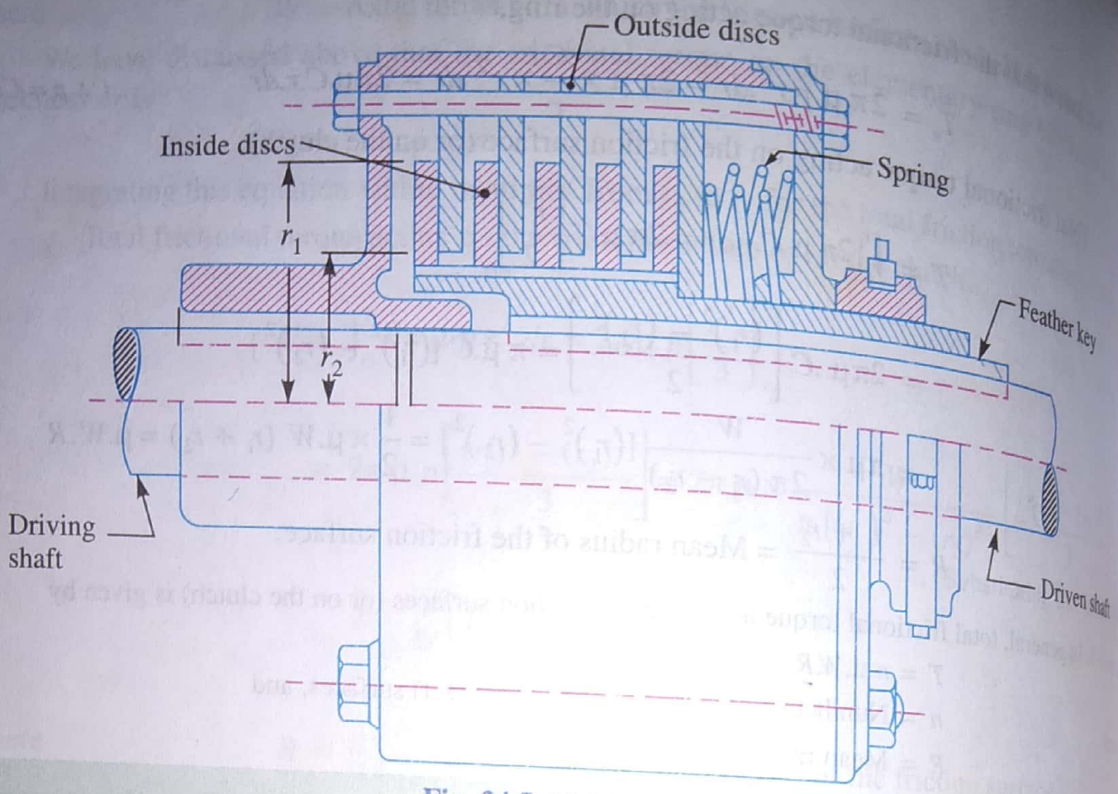


Fig. 24.5. Multiple disc clutch.

Let

n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where R = Mean radius of friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{... (For uniform wear)}$$

Example 24.1. *Determine the maximum, minimum and average pressure in a plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : W = 4 kN = 4000 N ; r_2 = 50 mm ; r_1 = 100 mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We also know that total force on the contact surface (W),

$$4000 = 2\pi C\,(r_1 - r_2) = 2\pi \times 50\, p_{max}\,(100 - 50) = 15\,710\, p_{max}$$

∴ $p_{max} = 4000 / 15\,710 = 0.2546 \text{ N/mm}^2$ **Ans.**

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore,

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface (W),

$$4000 = 2\pi C\,(r_1 - r_2) = 2\pi \times 100\, p_{min}\,(100 - 50) = 31\,420\, p_{min}$$

$$\therefore \quad p_{min} = 4000 / 31\,420 = 0.1273 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

$$= \frac{4000}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 24.2. *A plate clutch having a single driving plate with contact surfaces on each side is required to transmit 110 kW at 1250 r.p.m. The outer diameter of the contact surfaces is to be 300 mm. The coefficient of friction is 0.4.*

(a) Assuming a uniform pressure of 0.17 N/mm²; determine the inner diameter of the friction surfaces.

(b) Assuming the same dimensions and the same total axial thrust, determine the maximum torque that can be transmitted and the maximum intensity of pressure when uniform wear conditions have been reached.

Solution. Given : $P = 110$ kW $= 110 \times 10^3$ W ; $N = 1250$ r.p.m. ; $d_1 = 300$ mm or $r_1 = 150$ mm ; $\mu = 0.4$; $p = 0.17$ N/mm²

(a) Inner diameter of the friction surfaces

Let d_2 = Inner diameter of the contact or friction surfaces, and

r_2 = Inner radius of the contact or friction surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi\, N} = \frac{110 \times 10^3 \times 60}{2\,\pi \times 1250} = 840 \text{ N-m}$$

$$= 840 \times 10^3 \text{ N-mm}$$

Axial thrust with which the contact surfaces are held together,

$$W = \text{Pressure} \times \text{Area} = p \times \pi\,[(r_1)^2 - (r_2)^2]$$

$$= 0.17 \times \pi\,[(150)^2 - (r_2)^2] = 0.534\,[(150)^2 - (r_2)^2] \quad \ldots(i)$$

and mean radius of the contact surface for uniform pressure conditions,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right]$$

$\therefore$ Torque transmitted by the clutch (T),

$$840 \times 10^3 = n.\mu.W.R$$

$$= 2 \times 0.4 \times 0.534\,[(150)^2 - (r_2)^2] \times \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right] \quad \ldots(\because n = 2)$$

$$= 0.285\,[(150)^3 - (r_2)^3]$$

or

$$(150)^3 - (r_2)^3 = 840 \times 10^3 / 0.285 = 2.95 \times 10^6$$

$$\therefore \quad (r_2)^3 = (150)^3 - 2.95 \times 10^6 = 0.425 \times 10^6 \quad \text{or} \quad r_2 = 75 \text{ mm}$$

and

$$d_2 = 2r_2 = 2 \times 75 = 150 \text{ mm} \quad \textbf{Ans.}$$

(b) Maximum torque transmitted

We know that the axial thrust,

$$W = 0.534\,[(150)^2 - (r_2)^2]$$
$$= 0.534\,[(150)^2 - (75)^2] = 9011 \text{ N}$$ ... [From equation

and mean radius of the contact surfaces for uniform wear conditions,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 75}{2} = 112.5 \text{ mm}$$

∴ Maximum torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.4 \times 9011 \times 112.5 = 811 \times 10^3 \text{ N-mm}$$
$$= 811 \text{ N-m}$$ **Ans.**

Maximum intensity of pressure

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = p_{max} \times 75 \text{ N/mm}$$

We know that the axial thrust (W),

$$9011 = 2\,\pi\,C\,(r_1 - r_2) = 2\pi \times p_{max} \times 75\,(150 - 75) = 35\,347\,p_{max}$$

∴ $$p_{max} = 9011 / 35\,347 = 0.255 \text{ N/mm}^2$$ **Ans.**

Example 24.3. *A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner diameters of frictional surface if the coefficient of friction is 0.255, ratio of diameters is 1.25 and the maximum pressure is not to exceed 0.1 N/mm². Also, determine the axial thrust to be provided by springs. Assume the theory of uniform wear.*

Solution. Given : $n = 2$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 3000$ r.p.m. ; $\mu = 0.255$; $d_1 / d_2 = 1.25$ or $r_1 / r_2 = 1.25$; $p_{max} = 0.1$ N/mm²

Outer and inner diameters of frictional surface

Let d_1 and d_2 = Outer and inner diameters (in mm) of frictional surface, and

r_1 and r_2 = Corresponding radii (in mm) of frictional surface.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{25 \times 10^3 \times 60}{2\,\pi \times 3000} = 79.6 \text{ N-m} = 79\,600 \text{ N-mm}$$

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore.

$$p_{max} \times r_2 = C$$

or

$$C = 0.1\,r_2 \text{ N/mm}$$

and normal or axial load acting on the friction surface,

$$W = 2\pi\,C\,(r_1 - r_2) = 2\pi \times 0.1\,r_2\,(1.25\,r_2 - r_2)$$ ... ($\because r_1/r_2 = 1.25$)
$$= 0.157\,(r_2)^2$$

We know that mean radius of the frictional surface (for uniform wear),

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\,r_2 + r_2}{2} = 1.125\,r_2$$

and the torque transmitted (T),

$$79\,600 = n.\mu.W.R = 2 \times 0.255 \times 0.157\,(r_2)^2\,1.125\,r_2 = 0.09\,(r_2)^3$$

∴ $$(r_2)^3 = 79.6 \times 10^3 / 0.09 = 884 \times 10^3 \quad \text{or} \quad r_2 = 96 \text{ mm}$$

and $$r_1 = 1.25\,r_2 = 1.25 \times 96 = 120 \text{ mm}$$

∴ Outer diameter of frictional surface,

$$d_1 = 2r_1 = 2 \times 120 = 240 \text{ mm} \quad \textbf{Ans.}$$

and inner diameter of frictional surface,

$$d_2 = 2r_2 = 2 \times 96 = 192 \text{ mm} \quad \textbf{Ans.}$$

Axial thrust to be provided by springs

We know that axial thrust to be provided by springs,

$$W = 2\pi\, C\,(r_1 - r_2) = 2\pi \times 0.1\, r_2\,(1.25\, r_2 - r_2)$$

$$= 0.157\,(r_2)^2 = 0.157\,(96)^2 = 1447 \text{ N} \quad \textbf{Ans.}$$

Example 24.4. *A dry single plate clutch is to be designed for an automotive vehicle whose engine is rated to give 100 kW at 2400 r.p.m. and maximum torque 500 N-m. The outer radius of the friction plate is 25% more than the inner radius. The intensity of pressure between the plate is not to exceed 0.07 N/mm². The coefficient of friction may be assumed equal to 0.3. The helical springs required by this clutch to provide axial force necessary to engage the clutch are eight. If each spring has stiffness equal to 40 N/mm, determine the dimensions of the friction plate and initial compression in the springs.*

Solution. Given : P = 100 kW = 100 × 10³ W ; *N = 2400 r.p.m. ; T = 500 N-m = 500 × 10³ N-mm ; p = 0.07 N/mm² ; μ = 0.3 ; No. of springs = 8 ; Stiffness/spring = 40 N/mm

Dimensions of the friction plate

Let r_1 = Outer radius of the friction plate, and

r_2 = Inner radius of the friction plate.

Since the outer radius of the friction plate is 25% more than the inner radius, therefore

$$r_1 = 1.25\, r_2$$

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.07\, r_2 \text{ N/mm}$$

and axial load acting on the friction plate,

$$W = 2\pi\, C\,(r_1 - r_2) = 2\pi \times 0.07\, r_2\,(1.25\, r_2 - r_2) = 0.11\,(r_2)^2 \text{ N} \qquad \text{...}(i)$$

We know that mean radius of the friction plate, for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

∴ Torque transmitted (T),

$$500 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.11\,(r_2)^2\, 1.125\, r_2 = 0.074\,(r_2)^3 \qquad \text{...}(\because n = 2)$$

$$(r_2)^3 = 500 \times 10^3 / 0.074 = 6757 \times 10^3 \quad \text{or} \quad r_2 = 190 \text{ mm} \quad \textbf{Ans.}$$

and

$$r_1 = 1.25\, r_2 = 1.25 \times 190 = 237.5 \text{ mm} \quad \textbf{Ans.}$$

Initial compression in the springs

We know that total stiffness of the springs,

$$s = \text{Stiffness per spring} \times \text{No. of springs} = 40 \times 8 = 320 \text{ N/mm}$$

Axial force required to engage the clutch,

$$W = 0.11\,(r_2)^2 = 0.11\,(190)^2 = 3970 \text{ N} \qquad \text{... [From equation } (i)]$$

∴ Initial compression in the springs

$$= W / s = 3970 / 320 = 12.4 \text{ mm} \quad \textbf{Ans.}$$

* Superfluous data

and axial force required to engage the clutch,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 9.525 (125 - 75) = 2993 \text{ N}$$

Mean radius of the friction surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{125 + 75}{2} = 100 \text{ mm} = 0.1 \text{ m}$$

We know that the torque transmitted,

$$T = n.\mu.W.R = 4 \times 0.3 \times 2993 \times 0.1 = 359 \text{ N-m}$$

$\therefore$ Power transmitted, $P = \dfrac{T \times 2\pi N}{60} = \dfrac{359 \times 2\pi \times 500}{60} = 18\ 800 \text{ W} = 18.8 \text{ kW}$ **Ans.**

Example 24.8. *A multi-disc clutch has three discs on the driving shaft and two on the driven shaft. The inside diameter of the contact surface is 120 mm. The maximum pressure between the surface is limited to 0.1 N/mm². Design the clutch for transmitting 25 kW at 1575 r.p.m. Assume uniform wear condition and coefficient of friction as 0.3.*

Solution. Given : $n_1 = 3$; $n_2 = 2$; $d_2 = 120$ mm or $r_2 = 60$ mm ; $p_{max} = 0.1$ N/mm² ; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. ; $\mu = 0.3$

Let r_1 = Outside radius of the contact surface.

We know that the torque transmitted,

$$T = \frac{P \times 60}{2\pi N} = \frac{25 \times 10^3 \times 60}{2\pi \times 1575} = 151.6 \text{ N-m} = 151\ 600 \text{ N-mm}$$

For uniform wear, we know that $p.r = C$. Since the intensity of pressure is maximum at the inner radius (r_2), therefore,

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 0.1 \times 60 = 6 \text{ N/mm}$$

We know that the axial force on each friction surface,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 6 (r_1 - 60) = 37.7 (r_1 - 60) \quad ...(i)$$

For uniform wear, mean radius of the contact surface,

$$R = \frac{r_1 + r_2}{2} = \frac{r_1 + 60}{2} = 0.5\, r_1 + 30$$

We know that number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

$\therefore$ Torque transmitted (T),

$$151\ 600 = n.\mu.W.R = 4 \times 0.3 \times 37.7 (r_1 - 60)(0.5\, r_1 + 30)$$

... [Substituting the value of W from equation (i)]

$$= 22.62 (r_1)^2 - 81\ 432$$

$$\therefore \quad (r_1)^2 = \frac{151\ 600 + 81\ 432}{22.62} = 10\ 302$$

or $r_1 = 101.5$ mm **Ans.**

Example 24.9. *A multiple disc clutch, steel on bronze, is to transmit 4.5 kW at 750 r.p.m. The inner radius of the contact is 40 mm and outer radius of the contact is 70 mm. The clutch operates in oil with an expected coefficient of 0.1. The average allowable pressure is 0.35 N/mm². Find : 1. the total number of steel and bronze discs; 2. the actual axial force required; 3. the actual average pressure; and 4. the actual maximum pressure.*

Solution. Given : $P = 4.5$ kW $= 4500$ W ; $N = 750$ r.p.m. ; $r_2 = 40$ mm ; $r_1 = 70$ mm ; $\mu = 0.1$; $p_{av} = 0.35$ N/mm²

1. Total number of steel and bronze discs

Let n = Number of pairs of contact surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{4500 \times 60}{2\,\pi \times 750} = 57.3 \text{ N-m} = 57\,300 \text{ N-mm}$$

For uniform wear, mean radius of the contact surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{70 + 40}{2} = 55 \text{ mm}$$

and average axial force required,

$$W = p_{av} \times \pi\,[(r_1)^2 - (r_2)^2] = 0.35 \times \pi\,[(70)^2 - (40)^2] = 3630 \text{ N}$$

We also know that the torque transmitted (T),

$$57\,300 = n.\mu.W.R = n \times 0.1 \times 3630 \times 55 = 19\,965\,n$$

$$\therefore \quad n = 57\,300 / 19\,965 = 2.87$$

Since the number of pairs of contact surfaces must be even, therefore we shall use 4 pairs of contact surfaces with 3 steel discs and 2 bronze discs (because the number of pairs of contact surfaces is one less than the total number of discs). **Ans.**

2. Actual axial force required

Let W' = Actual axial force required.

Since the actual number of pairs of contact surfaces is 4, therefore actual torque developed by the clutch for one pair of contact surface,

$$T' = \frac{T}{n} = \frac{57\,300}{4} = 14\,325 \text{ N-mm}$$

We know that torque developed for one pair of contact surface (T'),

$$14\,325 = \mu.W'.R = 0.1 \times W' \times 55 = 5.5\,W'$$

$$\therefore \quad W' = 14\,325 / 5.5 = 2604.5 \text{ N} \quad \textbf{Ans.}$$

3. Actual average pressure

We know that the actual average pressure,

$$p'_{av} = \frac{W'}{\pi[(r_1)^2 - (r_2)^2]} = \frac{2604.5}{\pi\,[(70)^2 - (40)^2]} = 0.25 \text{ N/mm}^2 \quad \textbf{Ans.}$$

4. Actual maximum pressure

Let p_{max} = Actual maximum pressure.

For uniform wear, $p.r = C$. Since the intensity of pressure is maximum at the inner radius, therefore,

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 40\,p_{max} \text{ N/mm}$$

We know that the actual axial force (W'),

$$2604.5 = 2\pi C\,(r_1 - r_2) = 2\pi \times 40\,p_{max}\,(70 - 40) = 7541\,p_{max}$$

$$\therefore \quad p_{max} = 2604.5 / 7541 = 0.345 \text{ N/mm}^2 \quad \textbf{Ans.}$$

[U.Q Type]

Example 24.10. *A plate clutch has three discs on the driving shaft and two discs on the driven shaft, providing four pairs of contact surfaces. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform pressure and μ = 0.3, find the total spring load pressing the plates together to transmit 25 kW at 1575 r.p.m.*

If there are 6 springs each of stiffness 13 kN/m and each of the contact surfaces has worn away by 1.25 mm, find the maximum power that can be transmitted, assuming uniform wear.

Solution. Given : $n_1 = 3$; $n_2 = 2$; $n = 4$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m.

Total spring load

Let W = Total spring load.

We know that the torque transmitted,

$$T = \frac{P \times 60}{2\pi N} = \frac{25 \times 10^3 \times 60}{2\pi \times 1575} = 151.5 \text{ N-m}$$
$$= 151.5 \times 10^3 \text{ N-mm}$$

Mean radius of the contact surface, for uniform pressure,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(120)^3 - (60)^3}{(120)^2 - (60)^2}\right] = 93.3 \text{ mm}$$

and torque transmitted (T),

$$151.5 \times 10^3 = n.\mu.W.R = 4 \times 0.3 \times W \times 93.3 = 112\, W$$

$\therefore$ $W = 151.5 \times 10^3 / 112 = 1353$ N **Ans.**

Maximum power transmitted

Given : No. of springs = 6

$\therefore$ Contact surfaces of the spring = 8

Wear on each contact surface = 1.25 mm

$\therefore$ Total wear $= 8 \times 1.25 = 10$ mm $= 0.01$ m

Stiffness of each spring $= 13$ kN/m $= 13 \times 10^3$ N/m

$\therefore$ Reduction in spring force

= Total wear × Stiffness per spring × No. of springs

$= 0.01 \times 13 \times 10^3 \times 6 = 780$ N

and new axial load, $W = 1353 - 780 = 573$ N

We know that mean radius of the contact surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 60}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

and torque transmitted, $T = n \cdot \mu\, W \cdot R = 4 \times 0.3 \times 573 \times 0.09 = 62$ N-m

$\therefore$ Power transmitted, $$P = \frac{T \times 2\pi N}{60} = \frac{62 \times 2\pi \times 1575}{60} = 10\,227 \text{ W} = 10.227 \text{ kW}$$ **Ans.**

24.11 Cone Clutch

A cone clutch, as shown in Fig. 24.6, was extensively used in automobiles, but now-a-days it has been replaced completely by the disc clutch. It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at *B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

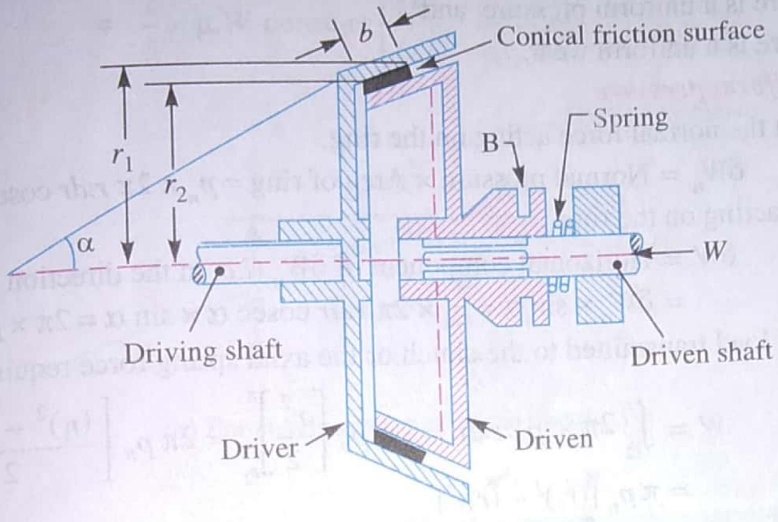


Fig. 24.6. Cone clutch.

24.12 Design of a Cone Clutch

Consider a pair of friction surfaces of a cone clutch as shown in Fig. 24.7. A little consideration will show that the area of contact of a pair of friction surface is a frustrum of a cone.

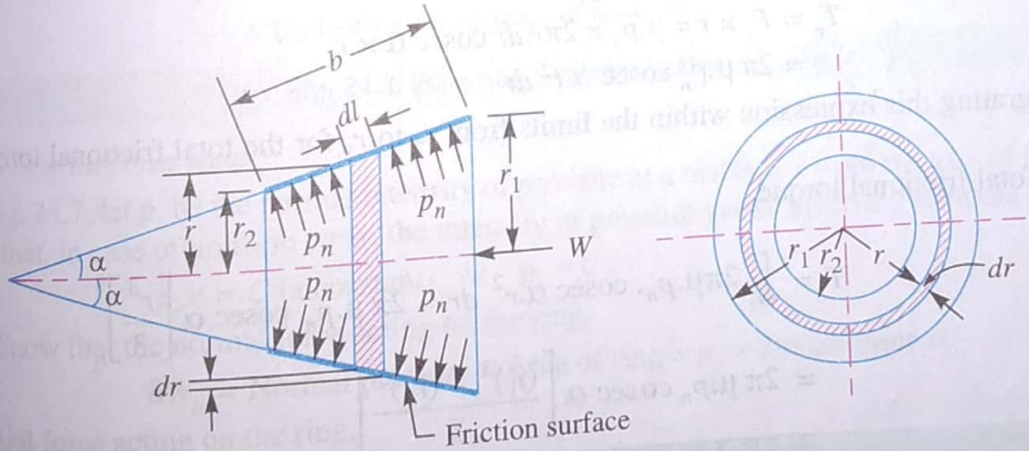


Fig. 24.7. Friction surfaces as a frustrum of a cone.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between the contact surfaces),

r_1 = Outer radius of friction surface,

r_2 = Inner radius of friction surface,

R = Mean radius of friction surface = $\dfrac{r_1 + r_2}{2}$,

α = Semi-angle of the cone (also called face angle of the cone) or angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between the contact surfaces, and

b = Width of the friction surfaces (also known as face width or cone face).

Consider a small ring of radius r and thickness dr as shown in Fig. 24.7. Let dl is the length of ring of the friction surface, such that,

$$dl = dr \text{ cosec } \alpha$$

$$\therefore \quad \text{Area of ring} = 2\pi\, r.\, dl = 2\pi\, r.dr \text{ cosec } \alpha$$

We shall now consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform wear.

1. Considering uniform pressure

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\pi\, r.dr \text{ cosec } \alpha$$

and the axial force acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \text{ (}i.e.\text{ in the direction of } W\text{)}$$

$$= \delta W_n \times \sin\alpha = p_n \times 2\pi\, r.dr \text{ cosec } \alpha \times \sin\alpha = 2\pi \times p_n.r.dr$$

$\therefore$ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi \times p_n.r.dr = 2\pi\ p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n\, [(r_1)^2 - (r_2)^2]$$

and

$$p_n = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\pi r.dr \text{ cosec } \alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\pi r.dr \text{ cosec } \alpha \times r$$

$$= 2\pi\, \mu.p_n \text{ cosec } \alpha.r^2\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

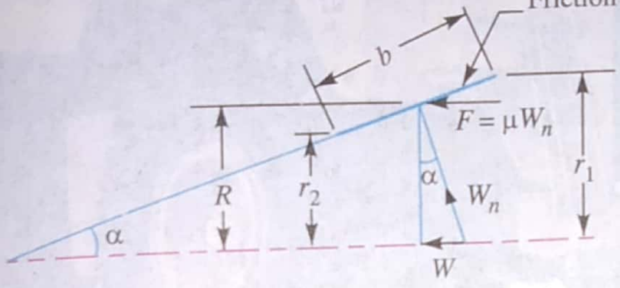
$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{ cosec } \alpha.r^2\ dr = 2\pi\ \mu.p_n \text{ cosec } \alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\ \mu.p_n \text{ cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

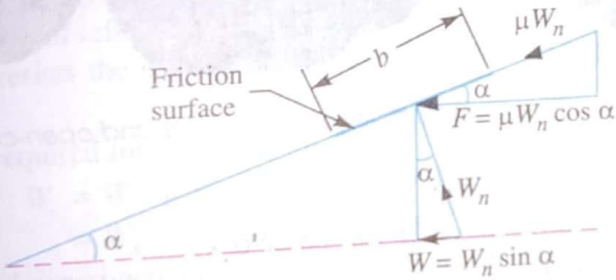
Substituting the value of p_n from equation (*i*), we get

$$T = 2\pi\,\mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]} \times \text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\ \text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad ...(ii)$$



(*a*) For steady operation of the clutch.



(*b*) During engagement of the clutch.

Fig. 24.8. Forces on a friction surface.

2. Considering uniform wear

In Fig. 24.7, let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$\therefore \quad p_r.r = C$ (a constant) or $p_r = C / r$

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr\ \text{cosec}\,\alpha$$

and the axial force acting on the ring,

$$\delta W = \delta W_n \times \sin\alpha = p_r \times 2\pi\ r.dr\ \text{cosec}\,\alpha \times \sin\alpha$$

$$= 2\pi \times p_r.r\ dr$$

$$= 2\pi \times \frac{C}{r} \times r.\ dr = 2\pi\ C.dr \quad ...\left(\because p_r = \frac{C}{r}\right)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi\ C.dr = 2\pi\ C\ [r]_{r_2}^{r_1} = 2\pi\ C\ (r_1 - r_2) \quad ...(iii)$$

or

$$C = \frac{W}{2\pi\ (r_1 - r_2)}$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\pi\ r.dr\ \text{cosec}\,\alpha$$

and inner radius of the clutch,

$$r_2 = R - \frac{b}{2}\sin\alpha = 250 - \frac{83.3}{2}\sin 12\frac{1}{2}° = 241 \text{ mm} \quad \textbf{Ans.}$$

24.13 Centrifugal Clutch

Centrifugal clutch with three discs and four steel float plates.

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 24.10. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The weight of the shoe, when revolving causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted.

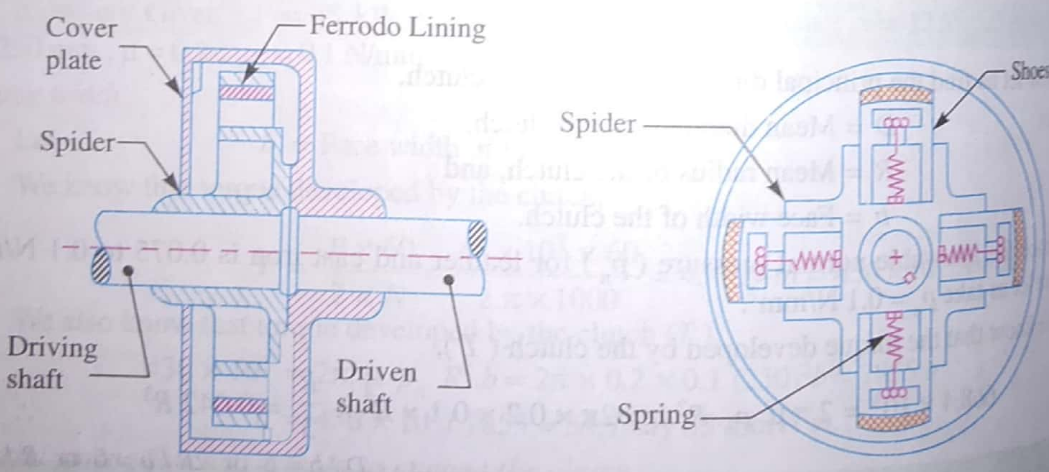


Fig. 24.10. Centrifugal clutch.

24.14 Design of a Centrifugal Clutch

In designing a centrifugal clutch, it is required to determine the weight of the shoe, size of the shoe and dimensions of the spring. The following procedure may be adopted for the design of a centrifugal clutch.

1. ***Mass of the shoes***

Consider one shoe of a centrifugal clutch as shown in Fig. 24.11.

Let $\quad m$ = Mass of each shoe,

$\quad n$ = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad / s = $2\pi N/60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

μ = Coefficient of friction between the shoe and rim.

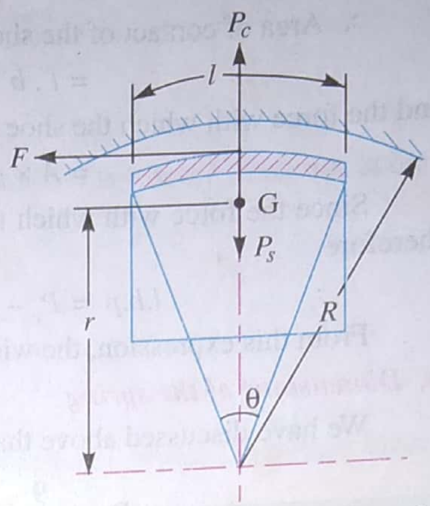


Fig. 24.11. Forces on a shoe of a centrifugal clucth.

We know that the centrifugal force acting on each shoe at the running speed,

$$*P_c = m.\omega^2.r$$

Since the speed at which the engagement begins to take place is generally taken as 3/4th of the running speed, therefore the inward force on each shoe exerted by the spring is given by

$$P_s = m\,(\omega_1)^2\,r = m\left(\frac{3}{4}\omega\right)^2 r = \frac{9}{16}\,m.\omega^2.r$$

$\therefore$ Net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s = m.\omega^2.r - \frac{9}{16}\,m.\omega^2.r = \frac{7}{16}\,m.\omega^2.r$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

$\therefore$ Frictional torque acting on each shoe

$$= F \times R = \mu\,(P_c - P_s)\,R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\,R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. Size of the shoes

Let l = Contact length of the shoes,

b = Width of the shoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley,

θ = Angle subtended by the shoes at the centre of the spider in radians, and

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, it may be taken as 0.1 N/mm².

We know that $\theta = \dfrac{l}{R}$ or $l = \theta.R = \dfrac{\pi}{3}R$ ...(Assuming $\theta = 60° = \pi/3$ rad)

* The radial clearance between the shoe and the rim is about 1.5 mm. Since this clearance is small as compared to r, therefore it is neglected for design purposes. If, however, the radial clearance is given, then the operating radius of the mass centre of the shoe from the axis of the clutch,

$$r_1 = r + c, \text{ where } c \text{ is the radial clearance,}$$

Then $P_c = m.\omega^2\,r_1$ and $P_s = m\,(\omega_1)^2\,r_1$

$\therefore$ Area of contact of the shoe

$$= l \,.\, b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

3. ***Dimensions of the spring***

We have discussed above that the load on the spring is given by

$$P_s = \frac{9}{16} \times m.\omega^2.r$$

The dimensions of the spring may be obtained as usual.

Example 24.14. *A centrifugal clutch is to be designed to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine: 1. mass of the shoes, and 2. size of the shoes.*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 900$ r.p.m. ; $n = 4$; $R = 150$ mm $= 0.15$ m ; $\mu = 0.25$

1. ***Mass of the shoes***

Let m = Mass of the shoes.

We know that the angular running speed,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

Since the speed at which the engagement begins is 3/4 th of the running speed, therefore angular speed at which engagement begins is

$$\omega_1 = \frac{3}{4}\,\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Assuming that the centre of gravity of the shoe lies at a distance of 120 mm (30 mm less than R) from the centre of the spider, *i.e.*

$$r = 120 \text{ mm} = 0.12 \text{ m}$$

We know that the centrifugal force acting on each shoe,

$$P_c = m.\omega^2.r = m\,(94.26)^2\, 0.12 = 1066\, m \text{ N}$$

and the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed, ω_1,

$$P_s = m(\omega_1)^2\, r = m\,(70.7)^2\, 0.12 = 600\, m \text{ N}$$

We know that the torque transmitted at the running speed,

$$T = \frac{P \times 60}{2\pi\, N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159 \text{ N-m}$$

We also know that the torque transmitted (T),

$$159 = \mu\,(P_c - P_s)\, R \times n = 0.25\,(1066\, m - 600\, m)\, 0.15 \times 4 = 70\, m$$

$$\therefore \quad m = 159/70 = 2.27 \text{ kg} \quad \textbf{Ans.}$$

Size of the shoes

Let l = Contact length of shoes in mm, and

b = Width of the shoes in mm.

Assuming that the arc of contact of the shoes subtend an angle of $\theta = 60°$ or $\pi / 3$ radians, at the centre of the spider, therefore

$$l = \theta.R = \frac{\pi}{3} \times 150 = 157 \text{ mm}$$

Area of contact of the shoes

$$A = l.b = 157\, b \text{ mm}^2$$

Assuming that the intensity of pressure (p) exerted on the shoes is 0.1 N/mm², therefore force with which the shoe presses against the rim

$$= A.p = 157b \times 0.1 = 15.7\, b \text{ N} \quad ...(i)$$

We also know that the force with which the shoe presses against the rim

$$= P_c - P_s = 1066\, m - 600\, m = 466\, m$$

$$= 466 \times 2.27 = 1058 \text{ N} \quad ...(ii)$$

From equations (*i*) and (*ii*), we find that

$$b = 1058 / 15.7 = 67.4 \text{ mm} \quad \textbf{Ans.}$$

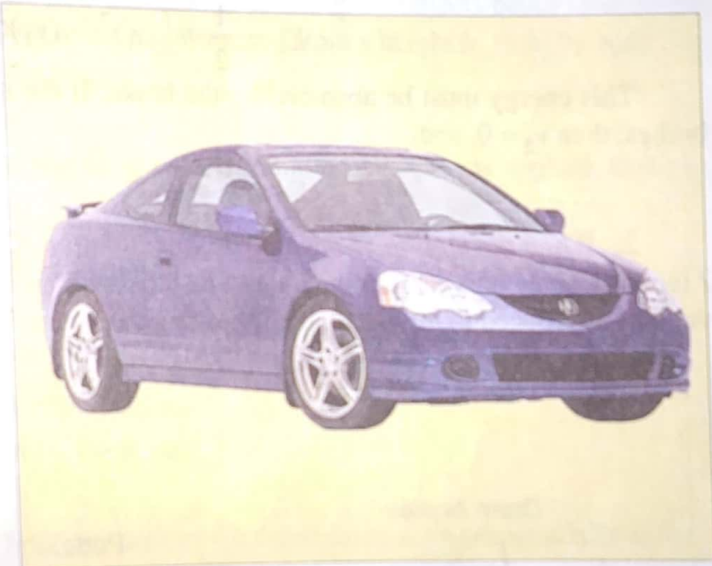
Special trailers are made to carry very long loads. The longest load ever moved was gas storage vessel, 83.8 m long.

EXERCISES

1. A single disc clutch with both sides of the disc effective is used to transmit 10 kW power at 900 r.p.m. The axial pressure is limited to 0.085 N/mm². If the external diameter of the friction lining is 1.25 times the internal diameter, find the required dimensions of the friction lining and the axial force exerted by the springs. Assume uniform wear conditions. The coefficient of friction may be taken as 0.3.
[Ans. 132.5 mm ; 106 mm ; 1500 N]

25

Brakes



25.1 Introduction

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air (or water which is circulated through the passages in the brake drum) so that excessive heating of the brake lining does not take place. The design or capacity of a brake depends upon the following factors :

1. The unit pressure between the braking surfaces,
2. The coefficient of friction between the braking surfaces,
3. The peripheral velocity of the brake drum,

4. The projected area of the friction surfaces, and
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

The major functional difference between a clutch and a brake is that a clutch is used to keep the driving and driven member moving together, whereas brakes are used to stop a moving member or to control its speed.

25.2 Energy Absorbed by a Brake

The energy absorbed by a brake depends upon the type of motion of the moving body. The motion of a body may be either pure translation or pure rotation or a combination of both translation and rotation. The energy corresponding to these motions is kinetic energy. Let us consider these motions as follows :

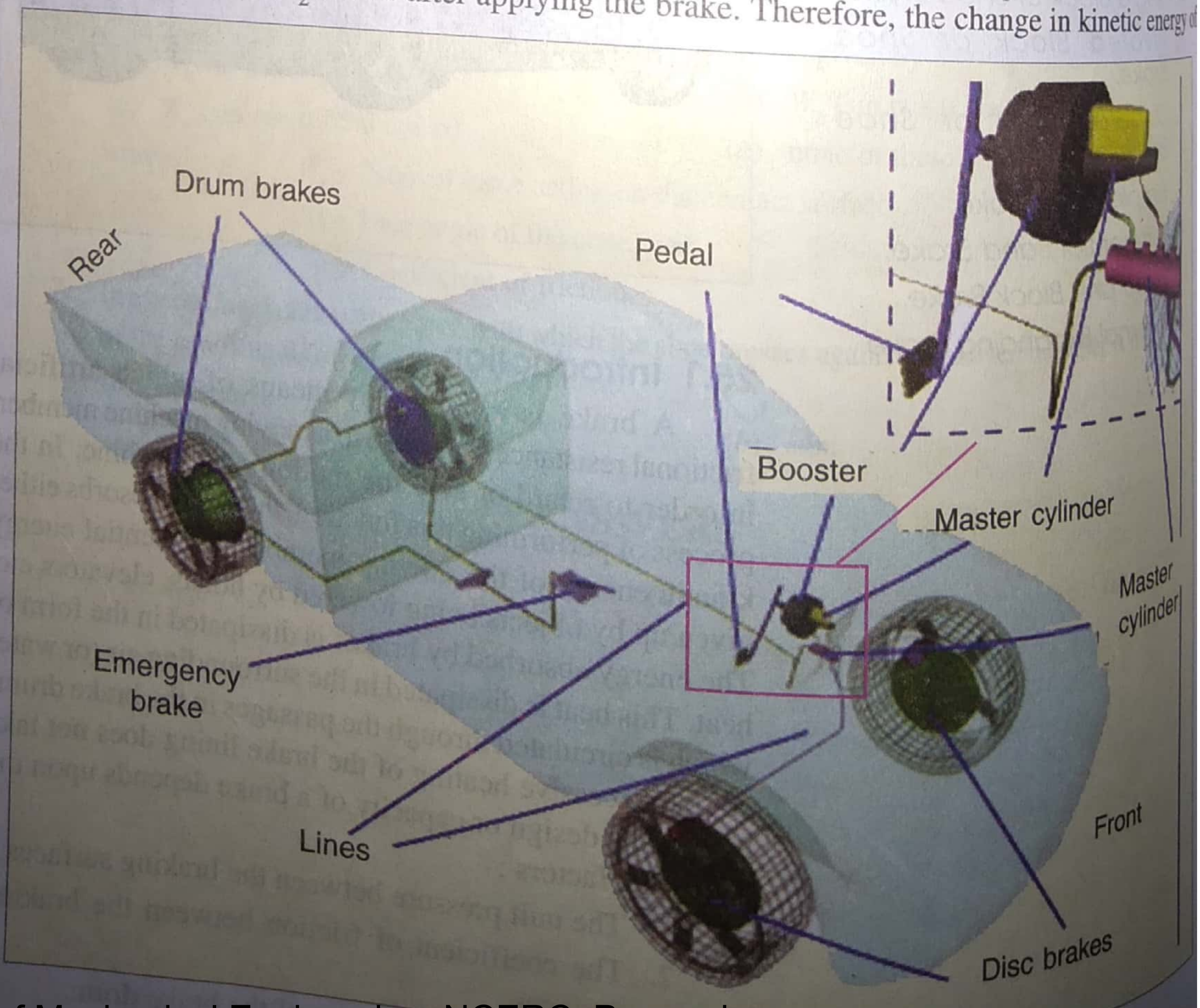
1. ***When the motion of the body is pure translation.*** Consider a body of mass (m) moving with a velocity v_1 m / s. Let its velocity is reduced to v_2 m / s by applying the brake. Therefore, the change in kinetic energy of the translating body or kinetic energy of translation,

$$E_1 = \frac{1}{2} m \left[(v_1)^2 - (v_2)^2 \right]$$

This energy must be absorbed by the brake. If the moving body is stopped after applying the brakes, then $v_2 = 0$, and

$$E_1 = \frac{1}{2} m (v_1)^2$$

2. ***When the motion of the body is pure rotation.*** Consider a body of mass moment of inertia I (about a given axis) is rotating about that axis with an angular velocity ω_1 rad / s. Let its angular velocity is reduced to ω_2 rad / s after applying the brake. Therefore, the change in kinetic energy of



the rotating body or kinetic energy of rotation,

$$E_2 = \frac{1}{2} I \left[(\omega_1)^2 - (\omega_2)^2 \right]$$

This energy must be absorbed by the brake. If the rotating body is stopped after applying the brakes, then $\omega_2 = 0$, and

$$E_2 = \frac{1}{2} I (\omega_1)^2$$

3. When the motion of the body is a combination of translation and rotation. Consider a body having both linear and angular motions, *e.g.* in the locomotive driving wheels and wheels of a moving car. In such cases, the total kinetic energy of the body is equal to the sum of the kinetic energies of translation and rotation.

∴ Total kinetic energy to be absorbed by the brake,

$$E = E_1 + E_2$$

Sometimes, the brake has to absorb the potential energy given up by objects being lowered by hoists, elevators etc. Consider a body of mass m is being lowered from a height h_1 to h_2 by applying the brake. Therefore the change in potential energy,

$$E_3 = m.g\,(h_1 - h_2)$$

If v_1 and v_2 m / s are the velocities of the mass before and after the brake is applied, then the change in potential energy is given by

$$E_3 = m.g \left(\frac{v_1 + v_2}{2} \right) t = m.g.v.t$$

where v = Mean velocity = $\frac{v_1 + v_2}{2}$, and

t = Time of brake application.

Thus, the total energy to be absorbed by the brake,

$$E = E_1 + E_2 + E_3$$

Let F_t = Tangential braking force or frictional force acting tangentially at the contact surface of the brake drum,

d = Diameter of the brake drum,

N_1 = Speed of the brake drum before the brake is applied,

N_2 = Speed of the brake drum after the brake is applied, and

N = Mean speed of the brake drum = $\frac{N_1 + N_2}{2}$

We know that the work done by the braking or frictional force in time t seconds

$$= F_t \times \pi\, d N \times t$$

Since the total energy to be absorbed by the brake must be equal to the wordone by the frictional force, therefore

$$E = F_t \times \pi\, d N \times t \quad \text{or} \quad F_t = \frac{E}{\pi d N . t}$$

The magnitude of F_t depends upon the final velocity (v_2) and on the braking time (t). Its value is maximum when $v_2 = 0$, *i.e.* when the load comes to rest finally.

We know that the torque which must be absorbed by the brake,

$$T = F_t \times r = F_t \times \frac{d}{2}$$

where r = Radius of the brake drum.

25.3 Heat to be Dissipated during Braking

The energy absorbed by the brake and transformed into heat must be dissipated to the surrounding air in order to avoid excessive temperature rise of the brake lining. The *temperature rise depends upon the mass of the brake drum, the braking time and the heat dissipation capacity of the brake. The highest permissible temperatures recommended for different brake lining materials are given as follows :

1. For leather, fibre and wood facing = 65 – 70°C
2. For asbestos and metal surfaces that are slightly lubricated = 90 – 105°C
3. For automobile brakes with asbestos block lining = 180 – 225°C

Since the energy absorbed (or heat generated) and the rate of wear of the brake lining at a particular speed are dependent on the normal pressure between the braking surfaces, therefore it is an important factor in the design of brakes. The permissible normal pressure between the braking surfaces depends upon the material of the brake lining, the coefficient of friction and the maximum rate at which the energy is to be absorbed. The energy absorbed or the heat generated is given by

$$E = H_g = \mu.R_N.v = \mu.p.A.v \text{ (in J/s or watts)} \quad ...(i)$$

where μ = Coefficient of friction,

R_N = Normal force acting at the contact surfaces, in newtons,

p = Normal pressure between the braking surfaces in N/m^2,

A = Projected area of the contact surfaces in m^2, and

v = Peripheral velocity of the brake drum in m/s.

The heat generated may also be obtained by considering the amount of kinetic or potential energies which is being absorbed. In other words,

$$H_g = E_K + E_P$$

where E_K = Total kinetic energy absorbed, and

E_P = Total potential energy absorbed.

The heat dissipated (H_d) may be estimated by

$$H_d = C\,(t_1 - t_2)\,A_r \quad ...(ii)$$

where C = Heat dissipation factor or coefficient of heat transfer in W /m^2/ °C

$t_1 - t_2$ = Temperature difference between the exposed radiating surface and the surrounding air in °C, and

A_r = Area of radiating surface in m^2.

The value of C may be of the order of 29.5 W / m^2 /°C for a temperature difference of 40°C and increase up to 44 W/m^2/°C for a temperature difference of 200°C.

The expressions for the heat dissipated are quite approximate and should serve only as an indication of the capacity of the brake to dissipate heat. The exact performance of the brake should be determined by test.

It has been found that 10 to 25 per cent of the heat generated is immediately dissipated to the surrounding air while the remaining heat is absorbed by the brake drum causing its temperature to rise. The rise in temperature of the brake drum is given by

$$\Delta t = \frac{H_g}{m.c} \quad ...(iii)$$

where Δt = Temperature rise of the brake drum in °C,

* When the temperature increases, the coefficient of friction decreases which adversely affect the torque capacity of the brake. At high temperature, there is a rapid wear of friction lining, which reduces the life of lining. Therefore, the temperature rise should be kept within the permissible range.

H_g = Heat generated by the brake in joules,

m = Mass of the brake drum in kg, and

c = Specific heat for the material of the brake drum in J/kg °C.

In brakes, it is very difficult to precisely calculate the temperature rise. In preliminary design analysis, the product $p.v$ is considered in place of temperature rise. The experience has also shown that if the product $p.v$ is high, the rate of wear of brake lining will be high and the brake life will be low. Thus the value of $p.v$ should be lower than the upper limit value for the brake lining to have reasonable wear life. The following table shows the recommended values of $p.v$ as suggested by various designers for different types of service.

Table 25.1. Recommended values of $p.v$.

S.No.	*Type of service*	*Recommended value of p.v in N-m/m² of projected area per second*
1.	Continuous application of load as in lowering operations and poor dissipation of heat.	0.98×10^6
2.	Intermittent application of load with comparatively long periods of rest and poor dissipation of heat.	1.93×10^6
3.	For continuous application of load and good dissipation of heat as in an oil bath.	2.9×10^6

Example 25.1. *A vehicle of mass 1200 kg is moving down the hill at a slope of 1: 5 at 72 km / h. It is to be stopped in a distance of 50 m. If the diameter of the tyre is 600 mm, determine the average braking torque to be applied to stop the vehicle, neglecting all the frictional energy except for the brake. If the friction energy is momentarily stored in a 20 kg cast iron brake drum, What is average temperature rise of the drum? The specific heat for cast iron may be taken as 520 J / kg°C.*

Determine, also, the minimum coefficient of friction between the tyres and the road in order that the wheels do not skid, assuming that the weight is equally distributed among all the four wheels.

Solution. Given : $m = 1200$ kg ; Slope = 1: 5 ; $v = 72$ km / h = 20 m/s ; $h = 50$ m ; $d = 600$ mm or $r = 300$ mm = 0.3 m ; $m_b = 20$ kg ; $c = 520$ J / kg°C

Average braking torque to be applied to stop the vehicle

We know that kinetic energy of the vehicle,

$$E_K = \frac{1}{2}\, m.v^2 = \frac{1}{2} \times 1200\,(20)^2 = 240\ 000 \text{ N-m}$$

and potential energy of the vehicle,

$$E_P = m.g.h \times \text{Slope} = 1200 \times 9.81 \times 50 \times \frac{1}{5} = 117\ 720 \text{ N-m}$$

∴ Total energy of the vehicle or the energy to be absorbed by the brake,

$$E = E_K + E_P = 240\ 000 + 117\ 720 = 357\ 720 \text{ N-m}$$

Since the vehicle is to be stopped in a distance of 50 m, therefore tangential braking force required,

$$F_t = 357\ 720 / 50 = 7154.4 \text{ N}$$

We know that average braking torque to be applied to stop the vehicle,

$$T_B = F_t \times r = 7154.4 \times 0.3 = 2146.32 \text{ N-m} \quad \textbf{Ans.}$$

Average temperature rise of the drum

Let Δt = Average temperature rise of the drum in °C.

We know that the heat absorbed by the brake drum,

$$H_g = \text{Energy absorbed by the brake drum}$$
$$= 357\,720 \text{ N-m} = 357\,720 \text{ J} \quad \text{... } (\because 1 \text{ N-m} = 1 \text{ J})$$

We also know that the heat absorbed by the brake drum (H_g),

$$357\,720 = m_b \times c \times \Delta t = 20 \times 520 \times \Delta t = 10\,400\,\Delta t$$

$$\therefore \quad \Delta t = 357\,720 / 10\,400 = 34.4\text{°C} \quad \textbf{Ans.}$$

Minimum coefficient of friction between the tyre and road

Let μ = Minimum coefficient of friction between the tyre and road, and

R_N = Normal force between the contact surface. This is equal to weight of the vehicle

$$= m.g = 1200 \times 9.81 = 11\,772 \text{ N}$$

We know that tangential braking force (F_t),

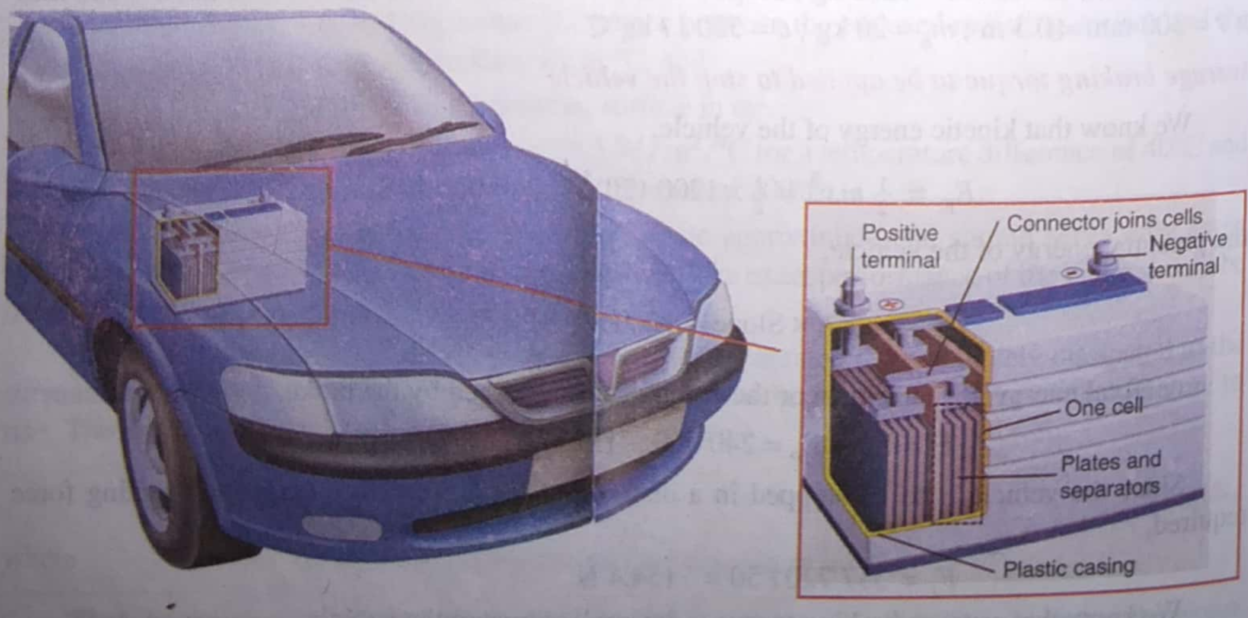
$$7154.4 = \mu.R_N = \mu \times 11\,772$$

$$\therefore \quad \mu = 7154.4 / 11772 = 0.6 \quad \textbf{Ans.}$$

25.4 Materials for Brake Lining

The material used for the brake lining should have the following characteristics :

1. It should have high coefficient of friction with minimum fading. In other words, the coefficient of friction should remain constant over the entire surface with change in temperature.
2. It should have low wear rate.
3. It should have high heat resistance.
4. It should have high heat dissipation capacity.
5. It should have low coefficient of thermal expansion.
6. It should have adequate mechanical strength.
7. It should not be affected by moisture and oil.



The rechargeable battery found in most cars is a combination of lead acid cells. A small dynamo, driven by the vehicle's engine, charges the battery whenever the engine is running.

The materials commonly used for facing or lining of brakes and their properties are shown in the following table.

Table 25.2. Properties of materials for brake lining.

Material for braking lining	Coefficient of friction (μ)			Allowable pressure (p)
	Dry	Greasy	Lubricated	N/mm^2
Cast iron on cast iron	0.15 – 0.2	0.06 – 0.10	0.05 – 0.10	1.0 – 1.75
Bronze on cast iron	–	0.05 – 0.10	0.05 – 0.10	0.56 – 0.84
Steel on cast iron	0.20 – 0.30	0.07 – 0.12	0.06 – 0.10	0.84 – 1.4
Wood on cast iron	0.20 – 0.35	0.08 – 0.12	–	0.40 – 0.62
Fibre on metal	–	0.10 – 0.20	–	0.07 – 0.28
Cork on metal	0.35	0.25 – 0.30	0.22 – 0.25	0.05 – 0.10
Leather on metal	0.3 – 0.5	0.15 – 0.20	0.12 – 0.15	0.07 – 0.28
Wire asbestos on metal	0.35 – 0.5	0.25 – 0.30	0.20 – 0.25	0.20 – 0.55
Asbestos blocks on metal	0.40 – 0.48	0.25 – 0.30	–	0.28 – 1.1
Asbestos on metal (Short action)	–	–	0.20 – 0.25	1.4 – 2.1
Metal on cast iron (Short action)	–	–	0.05 – 0.10	1.4 – 2.1

25.5 Types of Brakes

The brakes, according to the means used for transforming the energy by the braking element, are classified as :

1. Hydraulic brakes *e.g.* pumps or hydrodynamic brake and fluid agitator,
2. Electric brakes *e.g.* generators and eddy current brakes, and
3. Mechanical brakes.

Shoes of disk brakes of a racing car

The hydraulic and electric brakes cannot bring the member to rest and are mostly used where large amounts of energy are to be transformed while the brake is retarding the load such as in laboratory dynamometers, high way trucks and electric locomotives. These brakes are also used for retarding or controlling the speed of a vehicle for down-hill travel.

The mechanical brakes, according to the direction of acting force, may be divided into the following two groups :

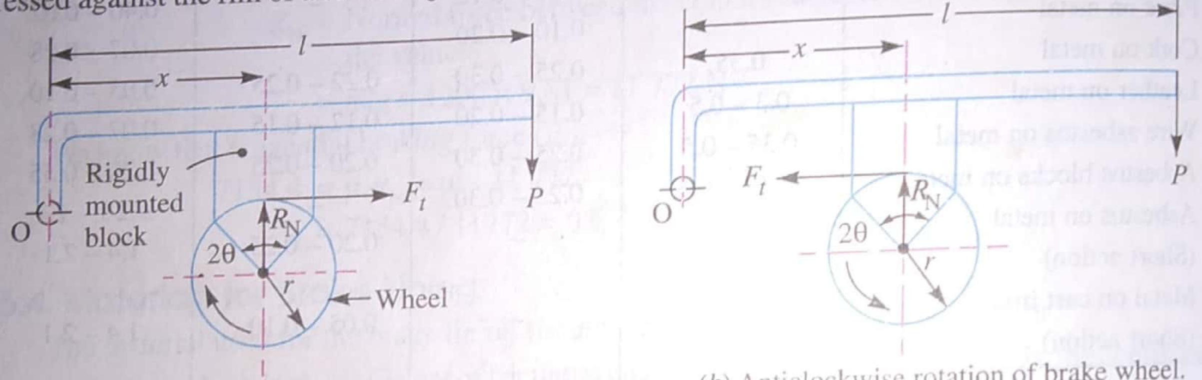
(a) ***Radial brakes.*** In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into ***external brakes*** and ***internal brakes.*** According to the shape of the friction element, these brakes may be block or shoe brakes and band brakes.

(b) ***Axial brakes.*** In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes. The analysis of these brakes is similar to clutches.

Since we are concerned with only mechanical brakes, therefore, these are discussed in detail, in the following pages.

25.6 Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. 25.1. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than



(a) Clockwise rotation of brake wheel.

(b) Anticlockwise rotation of brake wheel.

Fig. 25.1. Single block brake. Line of action of tangential force passes through the fulcrum of the lever.

the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. 25.1. The other end of the lever is pivoted on a fixed fulcrum *O*.

Let P = Force applied at the end of the lever,

R_N = Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu . R_N \quad \text{...(i)}$$

and the braking torque, $T_B = F_t . r = \mu\, R_N . r$ ...(ii)

Let us now consider the following three cases :

Case 1. When the line of action of tangential braking force (F_t) passes through the fulcrum *O* of the lever, and the brake wheel rotates clockwise as shown in Fig. 25.1 (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x = P \times l \quad \text{or} \quad R_N = \frac{P \times l}{x}$$

$\therefore$ Braking torque, $T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 25.1 (*b*), then the braking torque is same, *i.e.*

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case 2. When the line of action of the tangential braking force (F_t) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. 25.2 (*a*), then for equilibrium, taking moments about the fulcrum *O*,

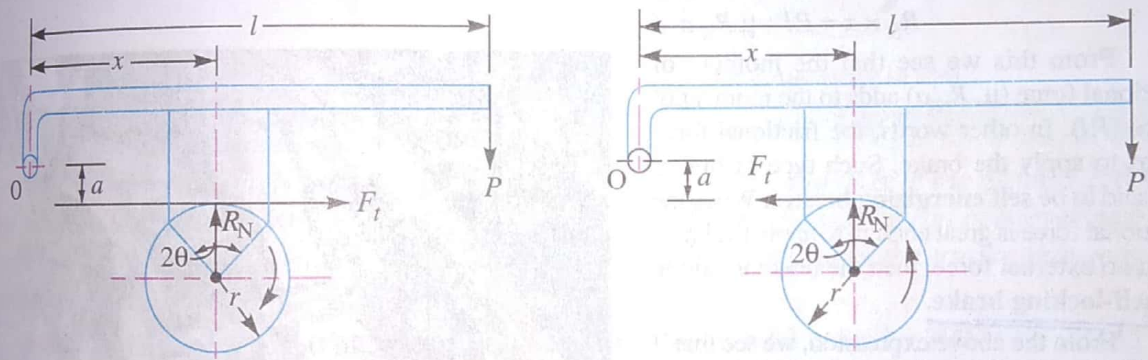
$$R_N \times x + F_t \times a = P.l$$

or

$$R_N \times x + \mu\, R_N \times a = P.l \qquad \text{or} \qquad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque,

$$T_B = \mu\, R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

Fig. 25.2. Single block brake. Line of action of F_t passes below the fulcrum.

When the brake wheel rotates anticlockwise, as shown in Fig. 25.2 (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \qquad \text{...(}i\text{)}$$

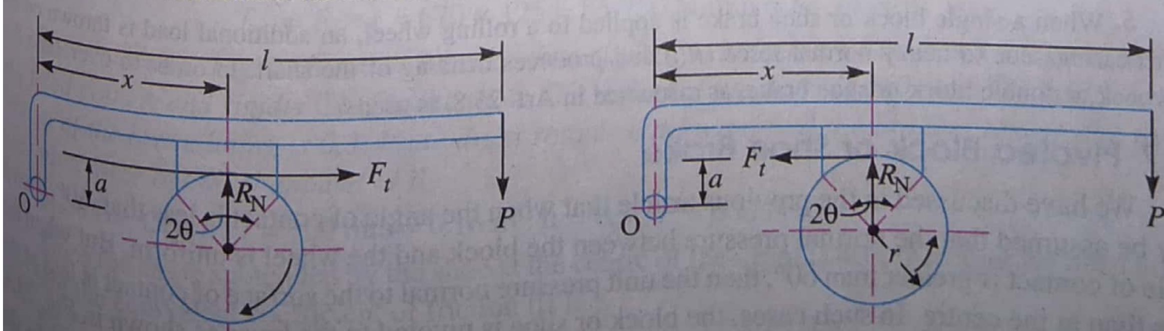
or

$$R_N\,(x - \mu.a) = P.l \qquad \text{or} \qquad R_N = \frac{P.l}{x - \mu.a}$$

and braking torque,

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

Case 3. When the line of action of the tangential braking force passes through a distance '*a*' above the fulcrum, and the brake wheel rotates clockwise as shown in Fig. 25.3 (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

Fig. 25.3. Single block brake. Line of action of F_t passes above the fulcrum.

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \quad ...(ii)$$

or $$R_N (x - \mu.a) = P.l \quad \text{or} \quad R_N = \frac{P.l}{x - \mu.a}$$

and braking torque, $$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

When the brake wheel rotates anticlockwise as shown in Fig. 25.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x + F_t \times a = P.l$$

or $$R_N \times x + \mu.R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque, $$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

Notes: 1. From above we see that when the brake wheel rotates anticlockwise in case 2 [Fig. 25.2 (*b*)] and when it rotates clockwise in case 3 [Fig. 25.3 (*a*)], the equations (*i*) and (*ii*) are same, *i.e.*

$$R_N \times x = P.l + \mu.R_N.a$$

From this we see that the moment of frictional force ($\mu.R_N.a$) adds to the moment of force ($P.l$). In other words, the frictional force helps to apply the brake. Such type of brakes are said to be **self energizing brakes.** When the frictional force is great enough to apply the brake with no external force, then the brake is said to be **self-locking brake.**

From the above expression, we see that if $x \le \mu.a$, then *P* will be negative or equal to zero. This means no external force is needed to apply the brake and hence the brake is self locking. Therefore the condition for the brake to be self locking is

$$x \le \mu.a$$

The self-locking brake is used only in back-stop applications.

Shoe of a bicycle

2. The brake should be self-energizing and not the self-locking.

3. In order to avoid self-locking and to prevent the brake from grabbing, *x* is kept greater than $\mu.a$.

4. If A_b is the projected bearing area of the block or shoe, then the bearing pressure on the shoe,

$$p_b = R_N / A_b$$

We know that A_b = Width of shoe × Projected length of shoe = w ($2r \sin \theta$)

5. When a single block or shoe brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to heavy normal force (R_N) and produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake, as discussed in Art. 25.8, is used.

25.7 Pivoted Block or Shoe Brake

We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre. In such cases, the block or shoe is pivoted to the lever as shown in Fig. 25.4, instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque for a pivoted block or shoe brake (*i.e.* when $2\theta > 60°$) is given by

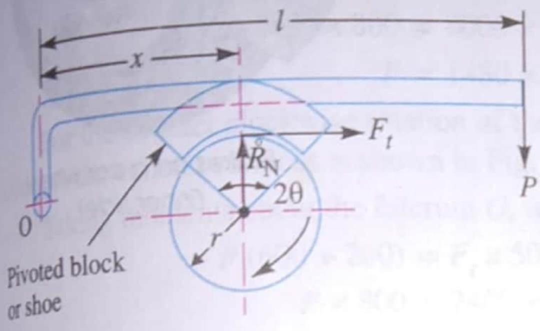
$$T_B = F_t \times r = \mu'.R_N.\,r$$

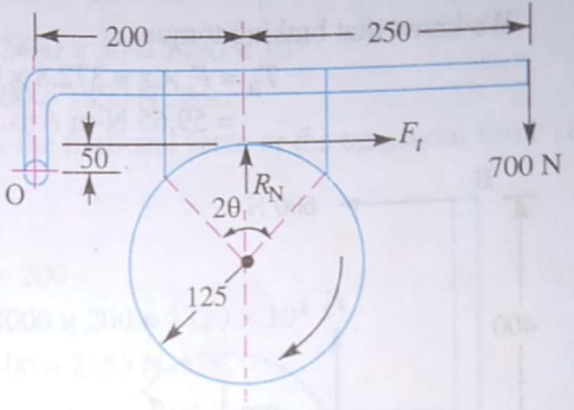
where

μ' = Equivalent coefficient of friction = $\dfrac{4\mu \sin\theta}{2\,\theta + \sin 2\theta}$, and

μ = Actual coefficient of friction.

These brakes have more life and may provide a higher braking torque.





All dimensions in mm.

Fig. 25.4. Pivoted block or shoe brake. **Fig. 25.5**

Example 25.2. *A single block brake is shown in Fig. 25.5. The diameter of the drum is 250 mm and the angle of contact is 90°. If the operating force of 700 N is applied at the end of a lever and the coefficient of friction between the drum and the lining is 0.35, determine the torque that may be transmitted by the block brake.*

Solution. Given : d = 250 mm or r = 125 mm ; $2\theta = 90° = \pi/2$ rad ; P = 700 N ; μ = 0.35

Since the angle of contact is greater than 60° , therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\,\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°} = 0.385$$

Let R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu'.\,R_N$

$F_t = \mu.R_N$

Taking moments above the fulcrum O, we have

$$700\,(250 + 200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\ F_t$$

or $520\,F_t - 50\,F_t = 700 \times 450$ or $F_t = 700 \times 450 / 470 = 670$ N

We know that torque transmitted by the block brake,

$T_B = F_t \times r = 670 \times 125 = 83\,750$ N-mm = 83.75 N-m **Ans.**

Example 25.3. *Fig. 25.6 shows a brake shoe applied to a drum by a lever AB which is pivoted at a fixed point A and rigidly fixed to the shoe. The radius of the drum is 160 mm. The coefficient of friction of the brake lining is 0.3. If the drum rotates clockwise, find the braking torque due to the horizontal force of 600 N applied at B.*

Solution. Given : r = 160 mm = 0.16 m ; μ = 0.3 ; P = 600 N

Since the angle subtended by the shoe at the centre of the drum is 40°, therefore we need not to calculate the equivalent coefficient of friction (μ').

Let R_N = Normal force pressing the shoe on the drum, and

F_t = Tangential braking force = $\mu.R_N$

Taking moments about point A,

$$R_N \times 350 + F_t\,(200 - 160) = 600\,(400 + 350)$$

$$\frac{F_t}{0.3} \times 350 + 40\,F_t = 600 \times 750$$

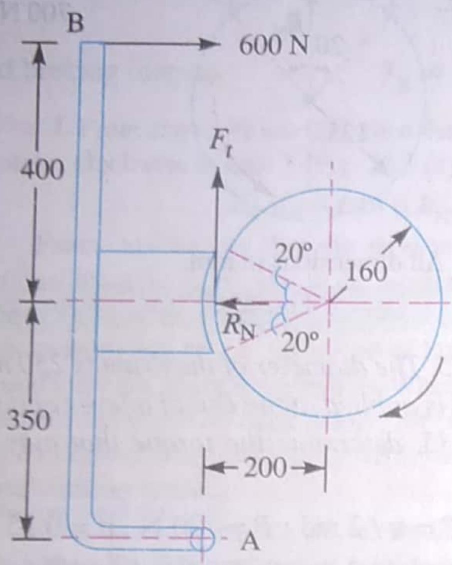
or $$207\,F_t = 450 \times 10^3$$

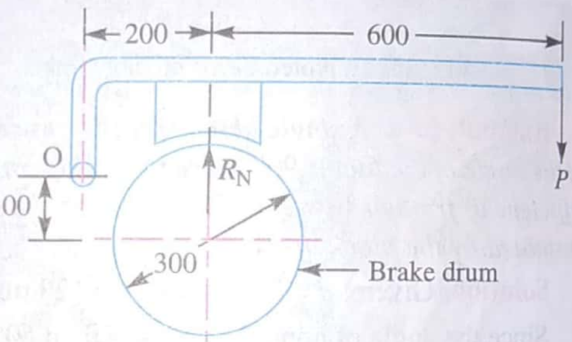
$$\therefore \quad F_t = 450 \times 10^3 / 1207 = 372.8 \text{ N}$$

We know that braking torque,

$$T_B = F_t \times r = 372.8 \times 0.16$$
$$= 59.65 \text{ N-m} \quad \textbf{Ans.}$$

Brakes on a car wheel (inner side)





All dimensions in mm.

Fig. 25.6 **Fig. 25.7**

Example 25.4. *The block brake, as shown in Fig. 25.7, provides a braking torque of 360 N-m. The diameter of the brake drum is 300 mm. The coefficient of friction is 0.3. Find :*

1. *The force (P) to be applied at the end of the lever for the clockwise and counter clockwise rotation of the brake drum; and*
2. *The location of the pivot or fulcrum to make the brake self locking for the clockwise rotation of the brake drum.*

Solution. Given : $T_B = 360$ N-m $= 360 \times 10^3$ N-mm ; $d = 300$ mm or $r = 150$ mm $= 0.15$ m ; $\mu = 0.3$

1. ***Force (P) for the clockwise and counter clockwise rotation of the brake drum***

For the clockwise rotation of the brake drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.8.

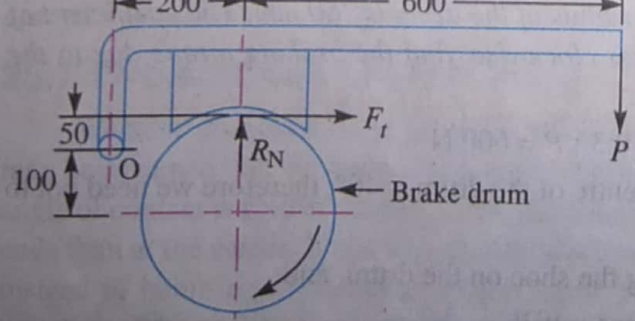


Fig. 25.8

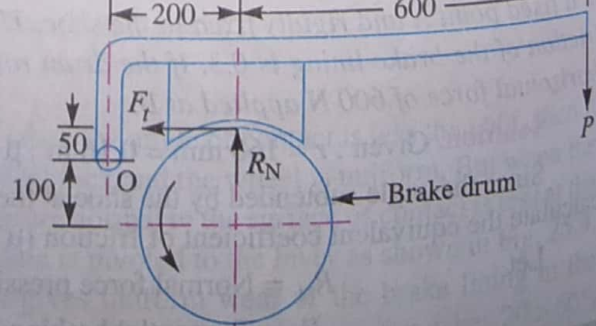


Fig. 25.9

We know that braking torque (T_B),

$$360 = F_t \times r = F_t \times 0.15 \quad \text{or} \quad F_t = 360 / 0.15 = 2400 \text{ N}$$

and normal force,

$$R_N = F_t / \mu = 2400 / 0.3 = 8000 \text{ N}$$

Now taking moments about the fulcrum O, we have

$$P(600 + 200) + F_t \times 50 = R_N \times 200$$

$$P \times 800 + 2400 \times 50 = 8000 \times 200$$

$$P \times 800 = 8000 \times 200 - 2400 \times 50 = 1480 \times 10^3$$

$$\therefore \quad P = 1480 \times 10^3 / 800 = 1850 \text{ N} \quad \textbf{Ans.}$$

For the counter clockwise rotation of the drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.9.

Taking moments about the fulcrum O, we have

$$P(600 + 200) = F_t \times 50 + R_N \times 200$$

$$P \times 800 = 2400 \times 50 + 8000 \times 200 = 1720 \times 10^3$$

$$\therefore \quad P = 1720 \times 10^3 / 800 = 2150 \text{ N} \quad \textbf{Ans.}$$

2. Location of the pivot or fulcrum to make the brake self-locking

The clockwise rotation of the brake drum is shown in Fig. 25.8. Let x be the distance of the pivot or fulcrum O from the line of action of the tangential force (F_t). Taking moments about the fulcrum O, we have

$$P(600 + 200) + F_t \times x - R_N \times 200 = 0$$

In order to make the brake self-locking, $F_t \times x$ must be equal to $R_N \times 200$ so that the force P is zero.

$$\therefore \quad F_t \times x = R_N \times 200$$

$$2400 \times x = 8000 \times 200 \quad \text{or} \quad x = 8000 \times 200 / 2400 = 667 \text{ mm} \quad \textbf{Ans.}$$

Example 25.5. *A rope drum of an elevator having 650 mm diameter is fitted with a brake drum of 1 m diameter. The brake drum is provided with four cast iron brake shoes each subtending an angle of 45°. The mass of the elevator when loaded is 2000 kg and moves with a speed of 2.5 m / s. The brake has a sufficient capacity to stop the elevator in 2.75 metres. Assuming the coefficient of friction between the brake drum and shoes as 0.2, find: 1. width of the shoe, if the allowable pressure on the brake shoe is limited to 0.3 N/mm²; and 2. heat generated in stopping the elevator.*

Solution. Given : $d_e = 650$ mm or $r_e = 325$ mm $= 0.325$ m ; $d = 1$ m or $r = 0.5$ m $= 500$ mm ; $n = 4$; $2\theta = 45°$ or $\theta = 22.5°$; $m = 2000$ kg ; $v = 2.5$ m / s ; $h = 2.75$ m ; $\mu = 0.2$; $p_b = 0.3$ N/mm²

1. Width of the shoe

Let w = Width of the shoe in mm.

First of all, let us find out the acceleration of the rope (a). We know that

$$v^2 - u^2 = 2\,a.h \quad \text{or} \quad (2.5)^2 - 0 = 2a \times 2.75 = 5.5a$$

$$\therefore \quad a = (2.5)^2 / 5.5 = 1.136 \text{ m/s}^2$$

and accelerating force = Mass × Acceleration $= m \times a = 2000 \times 1.136 = 2272$ N

∴ Total load acting on the rope while moving,

$$W = \text{Load on the elevator in newtons + Accelerating force}$$

$$= 2000 \times 9.81 + 2272 = 21\,892 \text{ N}$$

We know that torque acting on the shaft,

$$T = W \times r_e = 21\,892 \times 0.325 = 7115 \text{ N-m}$$

The brake drum is provided [illegible] therefore tangential force acting on each shoe,

$$F_t = 14\,230 / 4 = 3557.5 \text{ N}$$

Since the angle of contact of each shoe is 45°, therefore we need not to calculate the equivalent coefficient of friction (μ').

∴ Normal load on each shoe,

$$R_N = F_t / \mu = 3557.5 / 0.2 = 17\,787.5 \text{ N}$$

We know that the projected bearing area of each shoe,

$$A_b = w\,(2r \sin \theta) = w\,(2 \times 500 \sin 22.5°) = 382.7\, w \text{ mm}^2$$

We also know that bearing pressure on the shoe (p_b),

$$0.3 = \frac{R_N}{A_b} = \frac{17\,787.5}{382.7\, w} = \frac{46.5}{w}$$

$$\therefore \quad w = 46.5 / 0.3 = 155 \text{ mm} \quad \textbf{Ans.}$$

2. Heat generated in stopping the elevator

We know that heat generated in stopping the elevator

$$= \text{Total energy absorbed by the brake}$$

$$= \text{Kinetic energy} + \text{Potential energy} = \frac{1}{2} m.v^2 + m.g.h$$

$$= \frac{1}{2} \times 2000\,(2.5)^2 + 2000 \times 9.81 \times 2.75 = 60\,205 \text{ N-m}$$

$$= 60.205 \text{ kN-m} = 60.205 \text{ kJ} \quad \textbf{Ans.}$$

25.8 Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, and additional load is thrown on the shaft bearings due to the normal force (R_N). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake as shown in Fig. 25.10, is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force P is applied to the bell crank lever, the spring is compressed and the brake is released. This type of brake is often used on electric cranes and the force P is produced by an electromagnet or solenoid. When the current is switched off, there is no force on the bell crank lever and the brake is engaged automatically due to the spring force and thus there will be no downward movement of the load.

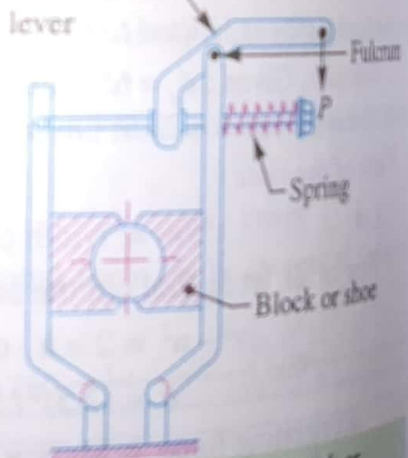


Fig. 25.10. Double block or shoe brake.

In a double block brake, the braking action is doubled by the use of two blocks and the two blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$$T_B = (F_{t1} + F_{t2})\, r$$

where F_{t1} and F_{t2} are the braking forces on the two blocks.

Example 25.6. *A double shoe brake, as shown in Fig. 25.11 is capable of absorbing a torque of 1400 N-m. The diameter of the brake drum is 350 mm and the angle of contact for each shoe is 100°. If the coefficient of friction between the brake drum and lining is 0.4; find : 1. the spring force*

The length of the spring in the closed position of the brake will be its free length. Assuming that the ends of the coil are squared and ground, therefore total number of coils,

$$n' = n + 2 = 8 + 2 = 10$$

∴ Free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta$$
$$= 10 \times 15 + 66 + 0.15 \times 66 = 226 \text{ mm } \textbf{Ans.}$$

3. Width of the brake shoes

Let b = Width of the brake shoes in mm, and

p_b = Bearing pressure on the lining material of the shoes.
$= 0.5 \text{ N/mm}^2$ ...(Given)

We know that projected bearing area for one shoe,

$$A_b = b\,(2r.\sin\theta) = b\,(2 \times 500 \sin 35°) = 574\,b \text{ mm}^2$$

We know that normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_{t1}}{\mu'} = \frac{0.59\,S}{0.32} = \frac{0.59 \times 4412}{0.32} = 8135 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_{t2}}{\mu'} = \frac{0.77\,S}{0.32} = \frac{0.77 \times 4412}{0.32} = 10\,616 \text{ N}$$

We see that the maximum normal force is on the left hand side of the shoe. Therefore we shall design the shoe for the maximum normal force *i.e.* R_{N2}.

We know that bearing pressure on the lining material (p_b),

$$0.5 = \frac{R_{N2}}{A_b} = \frac{10\,616}{574\,b} = \frac{18.5}{b}$$

$$\therefore \quad b = 18.5 / 0.5 = 37 \text{ mm } \textbf{Ans.}$$

4. Force required to be exerted by the thrustor to release the brake

Let P = Force required to be exerted by the thrustor to release the brake.

Taking moments about the fulcrum of the lever O, we have

$$P \times 500 + R_{N1} \times 650 = F_{t1}\,(500 - 250) + F_{t2}\,(500 + 250) + R_{N2} \times 650$$

$$P \times 500 + 8135 \times 650 = 0.59 \times 4412 + 250 + 0.77 \times 4412 \times 750 + 10\,616 \times 650$$

...(Substituting $F_{t1} = 0.59\,S$ and $F_{t2} = 0.77\,S$)

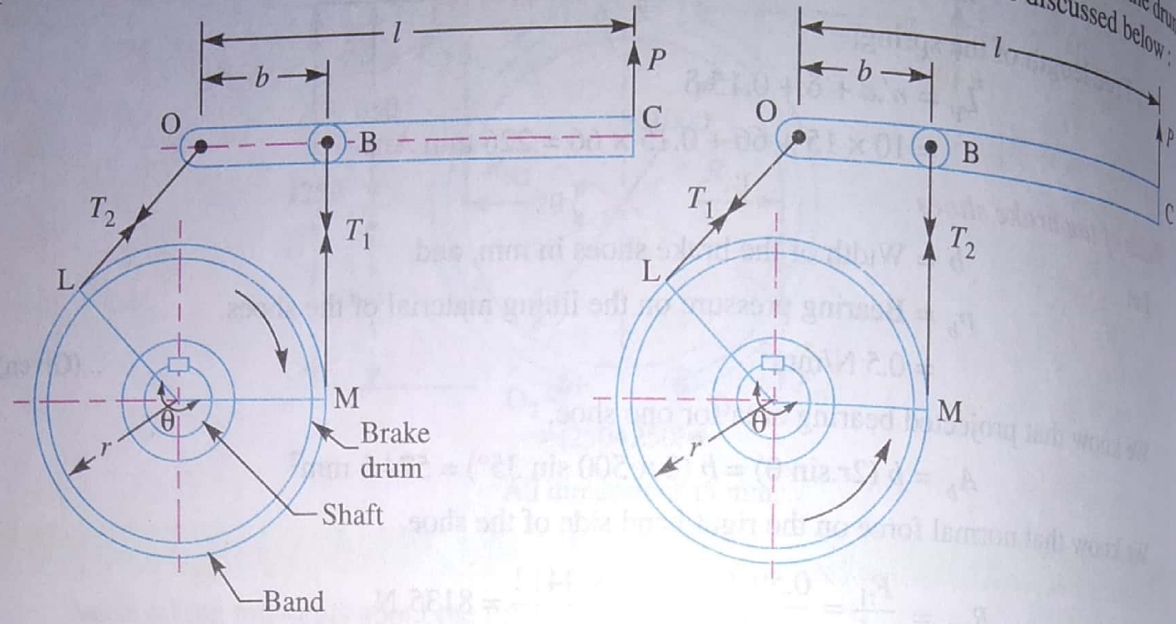
$$P \times 500 + 5.288 \times 10^6 = 0.65 \times 10^6 + 2.55 \times 10^6 + 6.9 \times 10^6 = 10.1 \times 10^6$$

$$\therefore \quad P = \frac{10.1 \times 10^6 - 5.288 \times 10^6}{500} = 9624 \text{ N } \textbf{Ans.}$$

25.9 Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig. 25.14, is called a ***simple band brake*** in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.

When a force P is applied to the lever at C, the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force P on the lever at C may be determined as discussed below :



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Fig. 25.14. Simple band brake.

Let T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum $= r + t/2$.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad 2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

and braking force on the drum

$$= T_1 - T_2$$

$\therefore$ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r \quad \text{...(Neglecting thickness of band)}$$

$$= (T_1 - T_2)\, r_e \quad \text{...(Considering thickness of band)}$$

Now considering the equilibrium of the lever OBC. It may be noted that when the drum rotates in the clockwise direction as shown in Fig. 25.14 (*a*), the end of the band attached to the fulcrum O will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction as shown in Fig. 25.14 (*b*), the tensions in the band will reverse, *i.e.* the end of the band attached to the fulcrum O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. Now taking moments about the fulcrum O, we have

$$P.l = T_1.b \quad \text{...(for clockwise rotation of the drum)}$$

and

$$P.l = T_2.b \quad \text{...(for anticlockwise rotation of the drum)}$$

where l = Length of the lever from the fulcrum (OC), and

b = Perpendicular distance from O to the line of action of T_1 or T_2.

Notes: **1.** When the brake band is attached to the lever, as shown in Fig. 25.14 (*a*) and (*b*), then the force (P) must act in the upward direction in order to tighten the band on the drum.

2. Sometimes the brake band is attached to the lever as shown in Fig. 25.15 (*a*) and (*b*), then the force (P) must act in the downward direction in order to tighten the band. In this case, for clockwise rotation of the drum, the end of the band attached to the fulcrum O will be tight with tension T_1 and band of the band attached to B will be slack with tension T_2. The tensions T_1 and T_2 will reverse for anticlockwise rotation of the drum.

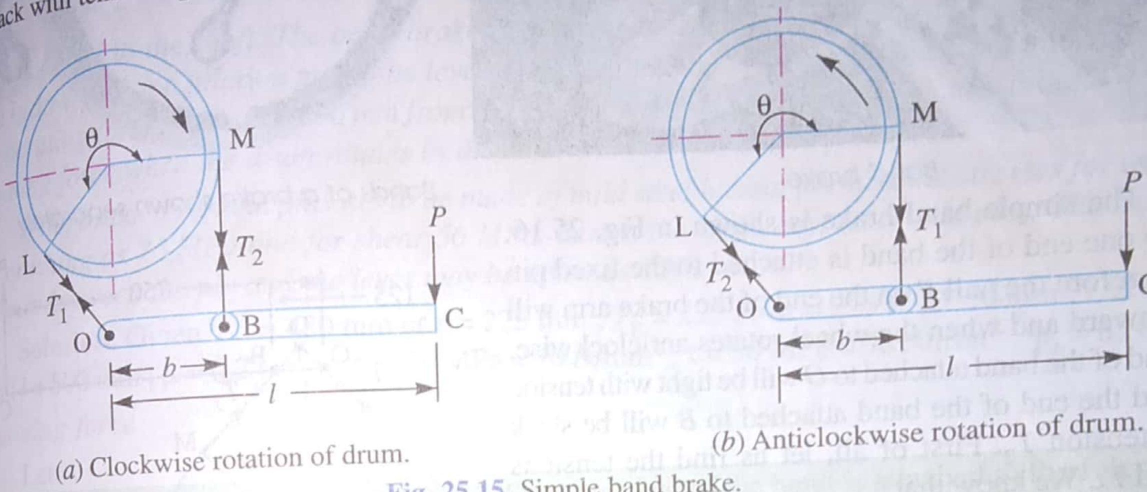


Fig. 25.15. Simple band brake.

3. If the permissible tensile stress (σ_t) for the material of the band is known, then maximum tension in the band is given by

$$T_1 = \sigma_t \times w \times t$$

where w = Width of the band, and

t = Thickness of the band.

4. The width of band (w) should not exceed 150 mm for drum diameter (d) greater than 1 metre and 100 mm for drum diameter less than 1 metre. The band thickness (t) may also be obtained by using the empirical relation *i.e.* $t = 0.005\ d$

For brakes of hand operated winches, the steel bands of the following sizes are usually used :

Width of band (w) in mm	25 – 40	40 – 60	80	100	140 – 200
Thickness of band (t) in mm	3	3 – 4	4 – 6	4 – 7	6 – 10

Example 25.8. *A simple band brake operates on a drum of 600 mm in diameter that is running at 200 r.p.m. The coefficient of friction is 0.25. The brake band has a contact of 270°, one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and placed perpendicular to the diameter that bisects the angle of contact.*

(*a*) *What is the pull necessary on the end of the brake arm to stop the wheel if 35 kW is being absorbed ? What is the direction for this minimum pull ?*

(*b*) *What width of steel band of 2.5 mm thick is required for this brake if the maximum tensile stress is not to exceed 50 MPa ?*

Solution. Given : d = 600 mm or r = 300 mm ; N = 200 r.p.m. ; μ = 0.25 ; θ = 270° = 270 × π/180 = 4.713 rad ; Power = 35 kW = 35 × 10³ W ; t = 2.5 mm ; σ_t = 50 MPa = 50 N/mm²

(*a*) ***Pull necessary on the end of the brake arm to stop the wheel***

Let P = Pull necessary on the end of the brake arm to stop the wheel.

Outer diameter of boss

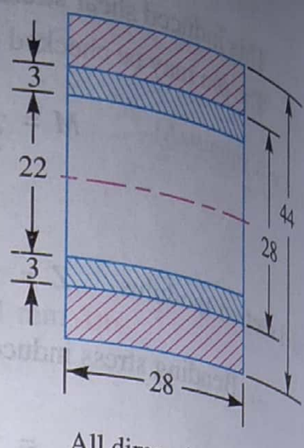
$= 2\,d_1 = 2 \times 22 = 44$ mm

= Outer diameter of eye

and thickness of each eye $= l_1/2 = 28/2 = 14$ mm

A clearance of 1.5 mm is provided on either side of the lever in the fork.

The new section of the lever at the fulcrum will be as shown in Fig. 25.32.



All dimensions in mm.

Fig. 25.32

∴ Section modulus,

$$Z = \frac{\frac{1}{12} \times 28\left[(44)^3 - (28)^3\right]}{44/2} = 6706 \text{ mm}^2$$

and induced bending stress $= \dfrac{431\,750}{6706} = 64.4$ N/mm^2

This induced bending stress is within permissible limits.

25.11 Band and Block Brake

The band brake may be lined with blocks of wood or other material, as shown in Fig. 25.33 (*a*). The friction between the blocks and the drum provides braking action. Let there are '*n*' number of blocks, each subtending an angle 2 θ at the centre and the drum rotates in anticlockwise direction.

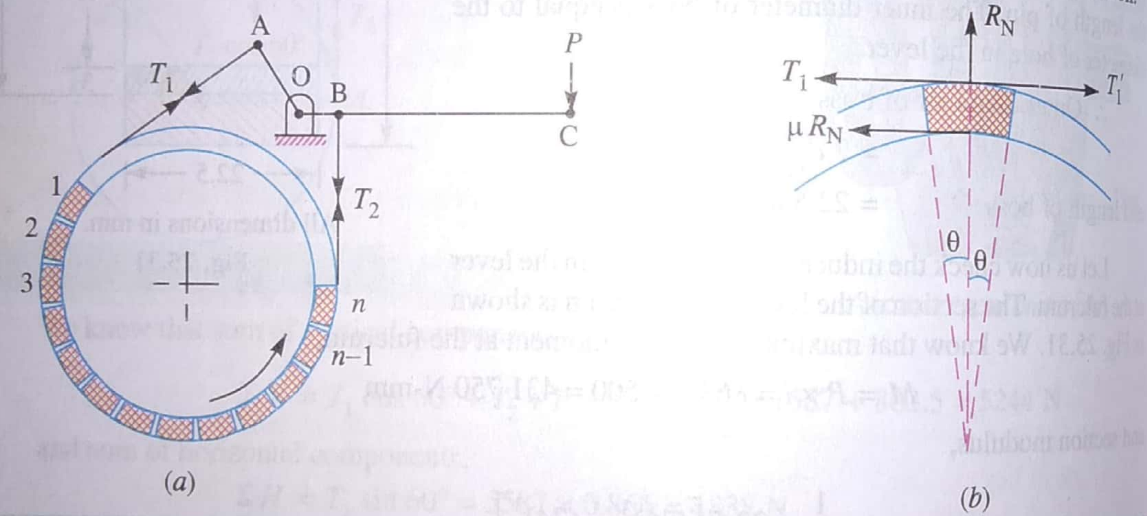


Fig. 25.33. Band and block brake.

Let T_1 = Tension in the tight side,

T_2 = Tension in the slack side,

μ = Coefficient of friction between the blocks and drum,

T_1' = Tension in the band between the first and second block,

T_2', T_3' etc. = Tensions in the band between the second and third block, between the third and fourth block etc.

Consider one of the blocks (say first block) as shown in Fig. 25.33 (*b*). This is in equilibrium under the action of the following forces :

1. Tension in the tight side (T_1),
2. Tension in the slack side (T_1') or tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction ($\mu.R_N$).

Resolving the forces radially, we have

$$(T_1 + T_1') \sin\theta = R_N \quad ...(i)$$

Resolving the forces tangentially, we have

$$(T_1 - T_1') \cos\theta = \mu . R_N \quad ...(ii)$$

Dividing equation *(ii)* by *(i)*, we have

$$\frac{(T_1 - T_1') \cos\theta}{(T_1 + T_1') \sin\theta} = \frac{\mu . R_N}{R_N}$$

or

$$(T_1 - T_1') = \mu \tan\theta \, (T_1 + T_1')$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}$$

Similarly it can be proved for each of the blocks that

$$\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'} = \frac{T_3'}{T_4'} = ... = \frac{T_{n-1}}{T_2} = \frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{T_1}{T_1'} \times \frac{T_1'}{T_2'} \times \frac{T_2'}{T_3'} \times ... \times \frac{T_{n-1}}{T_2} = \left(\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right)^n \quad ...(iii)$$

Braking torque on the drum of effective radius r_e,

$$T_B = (T_1 - T_2)\, r_e$$

$$= (T_1 - T_2)\, r \quad ...\text{(Neglecting thickness of band)}$$

Note: For the first block, the tension in the tight side is T_1 and in the slack side is T_1' and for the second block, the tension in the tight side is T_1' and in the slack side is T_2'. Similarly for the third block, the tension in the tight side is T_2' and in the slack side is T_3' and so on. For the last block, the tension in the tight side is T_{n-1} and in the slack side is T_2.

Example 25.14. *In the band and block brake shown in Fig. 25.34, the band is lined with 12 blocks each of which subtends an angle of 15° at the centre of the rotating drum. The thickness of the blocks is 75 mm and the diameter of the drum is 850 mm. If, when the brake is inaction, the greatest and least tensions in the brake strap are T_1 and T_2, show that*

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\,{}^1\!/_2{}^\circ}{1 - \mu \tan 7\,{}^1\!/_2{}^\circ}\right)^{12}$$

where μ is the coefficient of friction for the blocks.

With the lever arrangement as shown in Fig. 25.34, find the least force required at C for the blocks to absorb 225 kW at 240 r.p.m. The coefficient of friction between the band and blocks is 0.4.

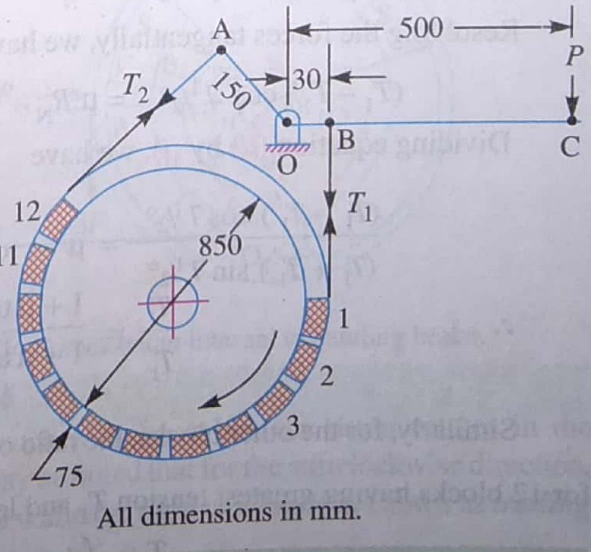


Fig. 25.34

Solution. Given : $n = 12$; $2\theta = 15°$ or $\theta = 7\,{}^1\!/_2°$; $t = 75$ mm $= 0.075$ m ; $d = 850$ mm $= 0.85$ m ; Power $= 225$ kW $= 225 \times 10^3$ W ; $N = 240$ r.p.m. ; $\mu = 0.4$

Since $OA > OB$, therefore the force at C must act downward. Also, the drum rotates clockwise, therefore the end of the band attached to A will be slack with tension T_2 (least tension) and the end of the band attached to B will be tight with tension T_1 (greatest tension).

Consider one of the blocks (say first block) as shown is Fig. 25.35. This is in equilibrium under the action of the following four forces :

1. Tension in the tight side (T_1),
2. Tension in the slack side (T_1') or the tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction ($\mu.R_N$).

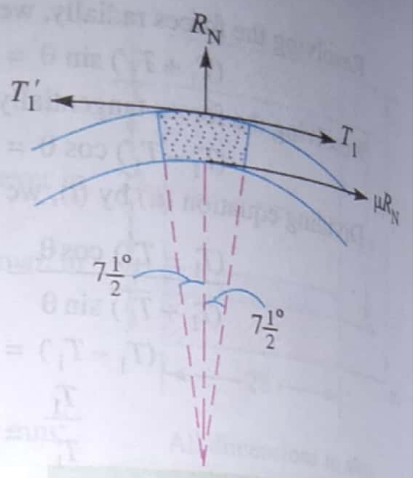
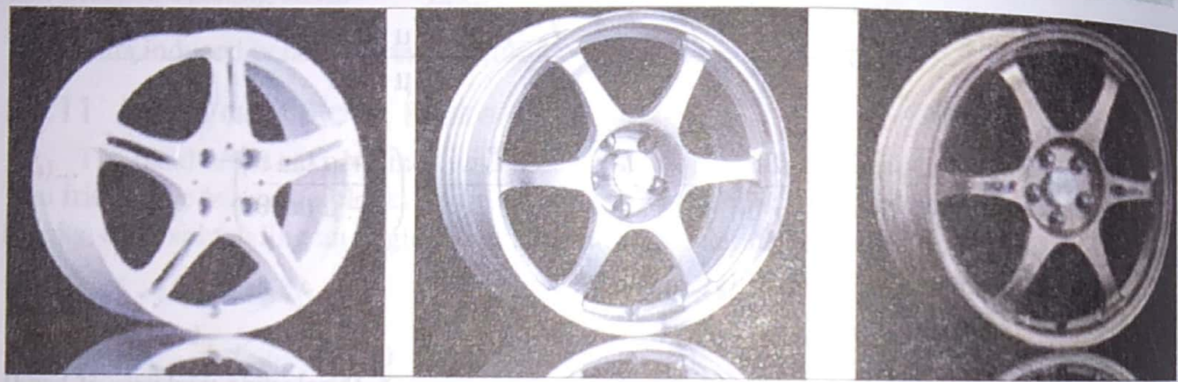


Fig. 25.35



Car wheels are made of alloys to bear high stresses and fatigue.

Resolving the forces radially, we have

$$(T_1 + T_1')\sin 7\,{}^1\!/_2° = R_N \qquad \text{...(i)}$$

Resolving the forces tangentially, we have

$$(T_1 - T_1')\cos 7\,{}^1\!/_2° = \mu.R_N \qquad \text{...(ii)}$$

Dividing equation *(ii)* by *(i)*, we have

$$\frac{(T_1 - T_1')\cos 7\,{}^1\!/_2°}{(T_1 + T_1')\sin 7\,{}^1\!/_2°} = \mu \quad \text{or} \quad \frac{T_1 - T_1'}{T_1 + T_1'} = \mu \tan 7\,{}^1\!/_2°$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1 + \mu \tan 7\,{}^1\!/_2°}{1 - \mu \tan 7\,{}^1\!/_2°}$$

Similarly, for the other blocks, the ratio of tensions $\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'}$ etc., remains constant. Therefore for 12 blocks having greatest tension T_1 and least tension T_2 is

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\,{}^1\!/_2°}{1 - \mu \tan 7\,{}^1\!/_2°}\right)^{12}$$

Least force required at C

Let P = Least force required at C.

We know that diameter of band,

$$D = d + 2\,t = 0.85 + 2 \times 0.075 = 1 \text{ m}$$

and power absorbed $= \dfrac{(T_1 - T_2)\pi D.N}{60}$

$$\therefore \quad T_1 - T_2 = \frac{\text{Power} \times 60}{\pi DN} = \frac{225 \times 10^3 \times 60}{\pi \times 1 \times 240} = 17\,900 \text{ N} \quad ...(iii)$$

We have proved that

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\,{}^1\!/_2°}{1 - \mu \tan 7\,{}^1\!/_2°}\right)^{12} = \left(\frac{1 + 0.4 \times 0.1317}{1 - 0.4 \times 0.1317}\right)^{12} = 3.55 \quad ...(iv)$$

From equations (*iii*) and (*iv*), we find that

$$T_1 = 24\,920 \text{ N} \text{ ; and } T_2 = 7020 \text{ N}$$

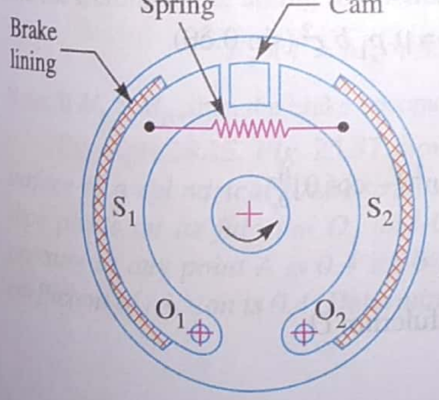
Now taking moments about O, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 30 = 7020 \times 150 - 24\,920 \times 30 = 305\,400$$

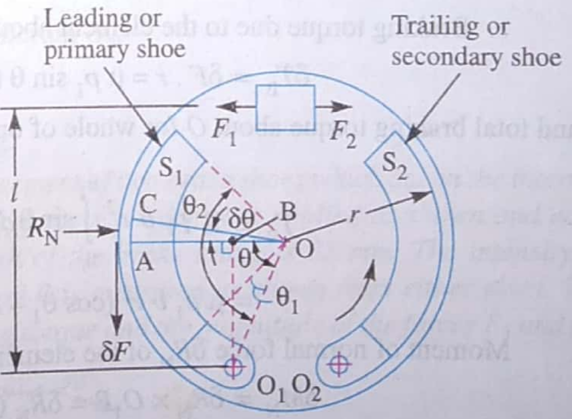
$$\therefore \quad P = 305\,400 / 500 = 610.8 \text{ N } \textbf{Ans.}$$

25.12 Internal Expanding Brake

An internal expanding brake consists of two shoes S_1 and S_2 as shown in Fig. 25.36 (*a*). The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum O_1 and O_2 and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. 25.36 (*a*). The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.



(*a*) Internal expanding brake.



(*b*) Forces on an internal expanding brake.

Fig. 25.36

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. 25.36 (*b*). It may be noted that for the anticlockwise direction, the left hand shoe is known as ***leading or primary shoe*** while the right hand shoe is known as ***trailing or secondary shoe.***

Let r = Internal radius of the wheel rim.

b = Width of the brake lining.

p_1 = Maximum intensity of normal pressure,

p_N = Normal pressure,

F_1 = Force exerted by the cam on the leading shoe, and

F_2 = Force exerted by the cam on the trailing shoe.

Inside view of a truck disk brake

Consider a small element of the brake lining AC subtending an angle $\delta\theta$ at the centre. Let OA makes an angle θ with OO_1 as shown in Fig. 25.36 (*b*). It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about O_1, therefore the rate of wear of the shoe lining at A will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from O_1 to OA, *i.e.* O_1B. From the geometry of the figure,

$$O_1B = OO_1 \sin\theta$$

and normal pressure at A, $p_N \propto \sin\theta$ or $p_N = p_1 \sin\theta$

∴ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N (b\,.\,r\,.\,\delta\theta) = p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)$$

and braking or friction force on the element,

$$\delta F = \mu.\delta R_N = \mu\, p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)$$

∴ Braking torque due to the element about O,

$$\delta T_B = \delta F\,.\,r = \mu\, p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)\,r = \mu\, p_1\, b\, r^2 (\sin\theta.\delta\theta)$$

and total braking torque about O for whole of one shoe,

$$T_B = \mu\, p_1\, b\, r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\, p_1\, b\, r^2\, [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1\, b\, r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1B = \delta R_N\,(OO_1 \sin\theta)$$

$$= p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)\,(OO_1 \sin\theta) = p_1 \sin^2\theta\,(b\,.\,r\,.\,\delta\theta)\,OO_1$$

Total moment of normal forces about the fulcrum O_1,

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta\,(b\,.\,r\,\delta\theta)\,OO_1 = p_1\,.\,b\,.\,r\,.\,OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

$$= p_1 . b . r . OO_1 \int_{\theta_1}^{\theta_2} \tfrac{1}{2}(1 - \cos 2\theta)\, d\theta \quad \ldots\left[\because \sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)\right]$$

$$= \tfrac{1}{2}\, p_1\,.\,b\,.\,r\,.\,OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2} p_1 \cdot b \cdot r \cdot OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \frac{1}{2} p_1 \cdot b \cdot r \cdot OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \times AB = \delta F\,(r - OO_1 \cos\theta) \qquad \ldots(\because AB = r - OO_1 \cos\theta)$$

$$= \mu \cdot p_1 \sin\theta\,(b \cdot r \cdot \delta\theta)\,(r - OO_1 \cos\theta)$$

$$= \mu \cdot p_1 \cdot b \cdot r\,(r \sin\theta - OO_1 \sin\theta\cos\theta)\,\delta\theta$$

$$= \mu \cdot p_1 \cdot b \cdot r \left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right)\delta\theta \qquad \ldots(\because 2\sin\theta\cos\theta = \sin 2\theta)$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu \cdot p_1 \cdot b \cdot r \int_{\theta_1}^{\theta_2}\left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right)d\theta$$

$$= \mu \cdot p_1 \cdot b \cdot r \left[-r\cos\theta + \frac{OO_1}{4}\cos 2\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu \cdot p_1 \cdot b \cdot r \left[-r\cos\theta_2 + \frac{OO_1}{4}\cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4}\cos 2\theta_1\right]$$

$$= \mu \cdot p_1 \cdot b \cdot r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

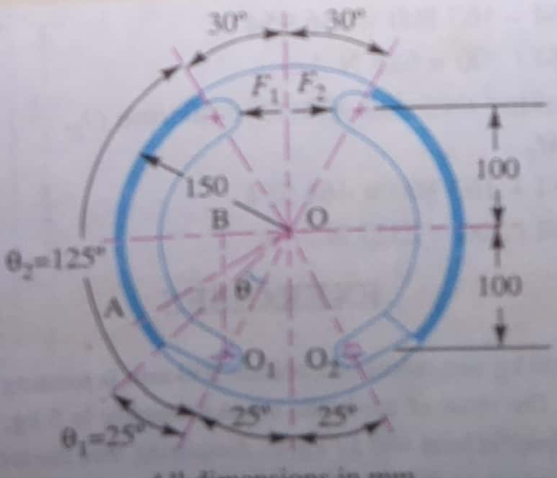
$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

Note: If $M_F > M_N$, then the brake becomes self locking.

Example 25.15. *Fig. 25.37 shows the arrangement of two brake shoes which act on the internal surface of a cylindrical brake drum. The braking force F_1 and F_2 are applied as shown and each shoe pivots on its fulcrum O_1 and O_2. The width of the brake lining is 35 mm. The intensity of pressure at any point A is 0.4 sin θ N/mm², where θ is measured as shown from either pivot. The coefficient of friction is 0.4. Determine the braking torque and the magnitude of the forces F_1 and F_2.*



All dimensions in mm.

Fig. 25.37

Solution. Given : $b = 35$ mm ; $\mu = 0.4$; $r = 150$ mm ; $l = 200$ mm ; $\theta_1 = 25°$; $\theta_2 = 125°$

Since the intensity of normal pressure at any point is $0.4 \sin \theta$ N/mm², therefore maximum intensity of normal pressure,

$$p_1 = 0.4 \text{ N/mm}^2$$

We know that the braking torque for one shoe,

$$= \mu \cdot p_1 \cdot b \cdot r^2 (\cos \theta_1 - \cos \theta_2)$$
$$= 0.4 \times 0.4 \times 35 (150)^2 (\cos 25° - \cos 125°)$$
$$= 126\,000 (0.9063 + 0.5736) = 186\,470 \text{ N-mm}$$

∴ Total braking torque for two shoes,

$$T_B = 2 \times 186\,470 = 372\,940 \text{ N-mm}$$

Magnitude of the forces F_1 and F_2

From the geometry of the figure, we find that

$$OO_1 = \frac{O_1B}{\cos 25°} = \frac{100}{0.9063} = 110.3 \text{ mm}$$
$$\theta_1 = 25° = 25 \times \pi / 180 = 0.436 \text{ rad}$$
and $$\theta_2 = 125° = 125 \times \pi / 180 = 2.18 \text{ rad}$$

We know that the total moment of normal forces about the fulcrum O_1,

$$M_N = \frac{1}{2} p_1 \cdot b \cdot r \cdot OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$
$$= \frac{1}{2} \times 0.4 \times 35 \times 150 \times 110.3 \left[(2.18 - 0.436) + \frac{1}{2}(\sin 50° - \sin 250°)\right]$$
$$= 115\,815 \left[1.744 + \frac{1}{2}(0.766 + 0.9397)\right] = 300\,754 \text{ N-mm}$$

and total moment of friction force about the fulcrum O_1,

$$M_F = \mu \cdot p_1 \cdot b \cdot r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$
$$= 0.4 \times 0.4 \times 35 \times 150 \left[150(\cos 25° - \cos 125°) + \frac{110.3}{4}(\cos 250° - \cos 50°)\right]$$
$$= 840 [150 (0.9063 + 0.5736) + 27.6 (-0.342 - 0.6428)]$$
$$= 840 (222 - 27) = 163\,800 \text{ N-mm}$$

For the leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$
or $$F_1 \times 200 = 300\,754 - 163\,800 = 136\,954$$
∴ $$F_1 = 136\,954 / 200 = 685 \text{ N}$$ **Ans.**

For the trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$
or $$F_2 \times 200 = 300\,754 + 163\,800 = 464\,554$$
∴ $$F_2 = 464\,554 / 200 = 2323 \text{ N}$$ **Ans.**

EXERCISES

1. A flywheel of mass 100 kg and radius of gyration 350 mm is rotating at 720 r.p.m. It is brought to rest by means of a brake. The mass of the brake drum assembly is 5 kg. The brake drum is made of cast iron FG 260 having specific heat 460 J / kg°C. Assuming that the total heat generated is absorbed by the brake drum only, calculate the temperature rise. [Ans. 15.8°C]

C H A P T E R

26

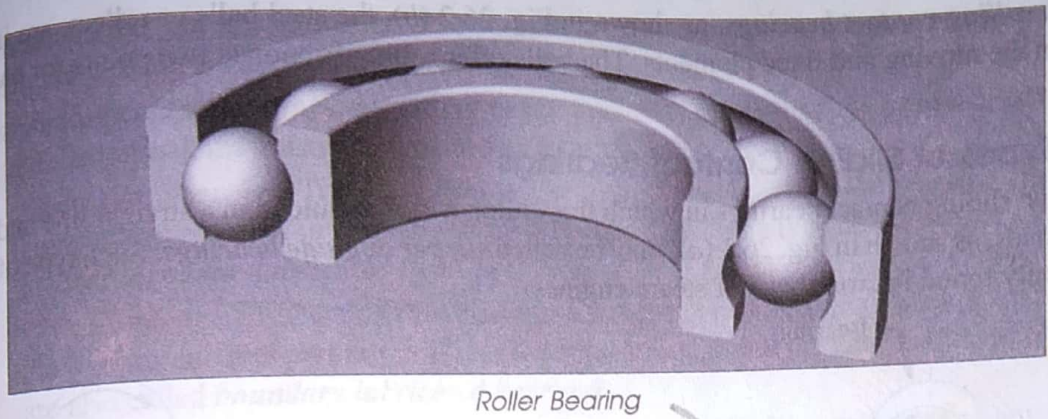
Sliding Contact Bearings

26.1 Introduction

A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may be used.

26.2 Classification of Bearings

Though the bearings may be classified in many ways, yet the following are important from the subject point of view:



Roller Bearing

1. *Depending upon the direction of load to be supported.* The bearings under this group are classified as:

(*a*) Radial bearings, and (*b*) Thrust bearings.

In ***radial bearings***, the load acts perpendicular to the direction of motion of the moving element as shown in Fig. 26.1 (*a*) and (*b*).

In ***thrust bearings,*** the load acts along the axis of rotation as shown in Fig. 26.1 (*c*).

Note : These bearings may move in either of the directions as shown in Fig. 26.1.

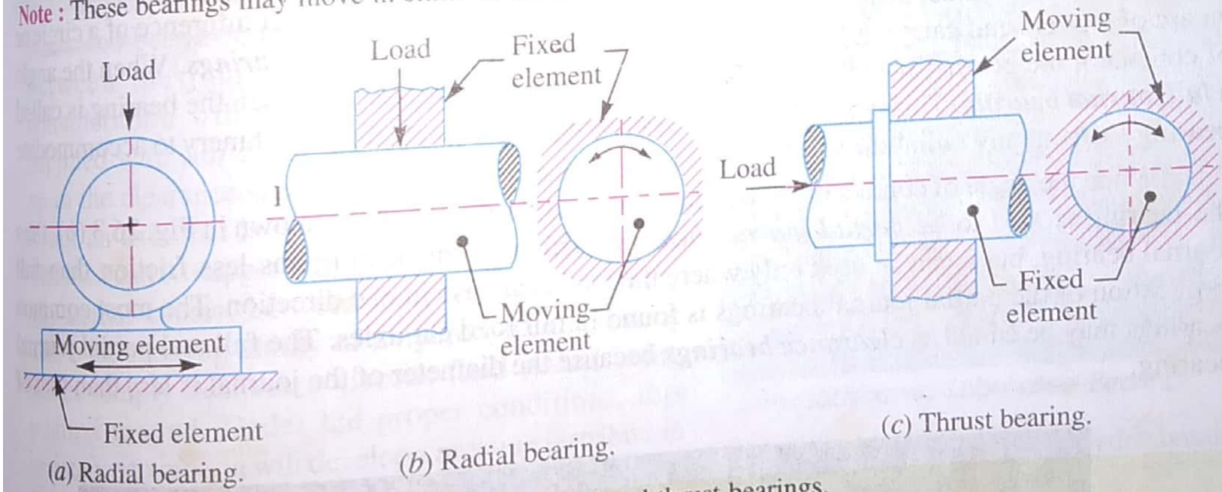


Fig. 26.1. Radial and thrust bearings.

2. *Depending upon the nature of contact.* The bearings under this group are classified as :

(*a*) Sliding contact bearings, and (*b*) Rolling contact bearings.

In ***sliding contact bearings,*** as shown in Fig. 26.2 (*a*), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as ***plain bearings.***

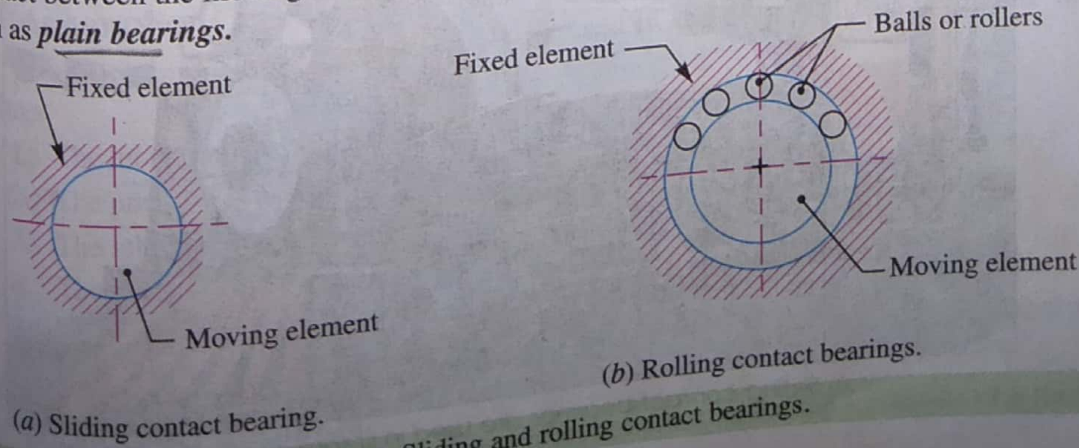
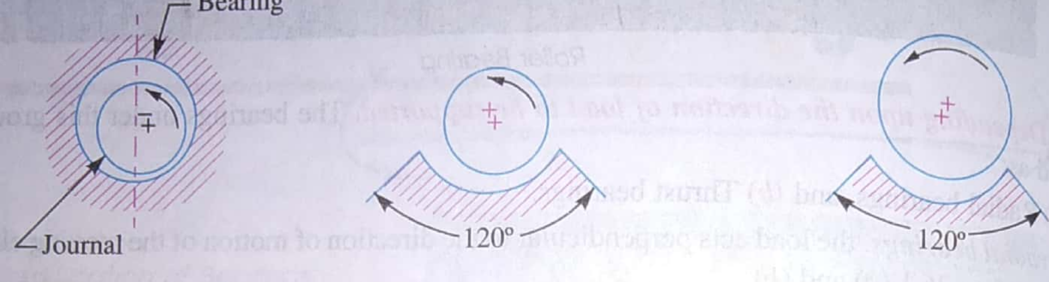


Fig. 26.2. Sliding and rolling contact bearings.

In ***rolling contact bearings,*** as shown in Fig. 26.2 (*b*), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.

26.3 Types of Sliding Contact Bearings

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig. 26.1 (*a*), may be called ***slipper*** or ***guide bearings.*** Such type of bearings are usually found in cross-head of steam engines.



(*a*) Full journal bearing. (*b*) Partial journal bearing. (*c*) Fitted journal bearing.

Fig. 26.3. Journal or sleeve bearings.

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as ***journal*** or ***sleeve bearings.*** When the angle of contact of the bearing with the journal is 360° as shown in Fig. 26.3 (*a*), then the bearing is called a ***full journal bearing.*** This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.

When the angle of contact of the bearing with the journal is 120°, as shown in Fig. 26.3 (*b*), then the bearing is said to be ***partial journal bearing.*** This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as ***clearance bearings*** because the diameter of the journal is less than that of bearing.

Sliding contact bearings are used in steam engines

When a partial journal bearing has no clearance *i.e.* the diameters of the journal and bearing are equal, then the bearing is called a ***fitted bearing***, as shown in Fig. 26.3 (*c*).

The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows :

1. *Thick film bearings*. The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as ***hydrodynamic lubricated bearings***.
2. *Thin film bearings*. The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other atleast part of the time. Such type of bearings are also called ***boundary lubricated bearings***.
3. *Zero film bearings*. The zero film bearings are those which operate without any lubricant present.
4. *Hydrostatic or externally pressurized lubricated bearings*. The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

26.4 Hydrodynamic Lubricated Bearings

Hydrodynamic Lubricated Bearings

We have already discussed that in hydrodynamic lubricated bearings, there is a thick film of lubricant between the journal and the bearing. A little consideration will show that when the bearing is supplied with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is eccentric with the bearing axis. The load can be supported by this fluid pressure without any actual contact between the journal and bearing. The load carrying ability of a hydrodynamic bearing arises simply because a viscous fluid resists being pushed around. Under the proper conditions, this resistance to motion will develop a pressure distribution in the lubricant film that can support a useful load. The load supporting pressure in hydrodynamic bearings arises from either

1. the flow of a viscous fluid in a converging channel (known as ***wedge film lubrication***), or
2. the resistance of a viscous fluid to being squeezed out from between approaching surfaces (known as ***squeeze film lubrication***).

26.5 Assumptions in Hydrodynamic Lubricated Bearings

The following are the basic assumptions used in the theory of hydrodynamic lubricated bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.
4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional, *i.e.* the side leakage is neglected.

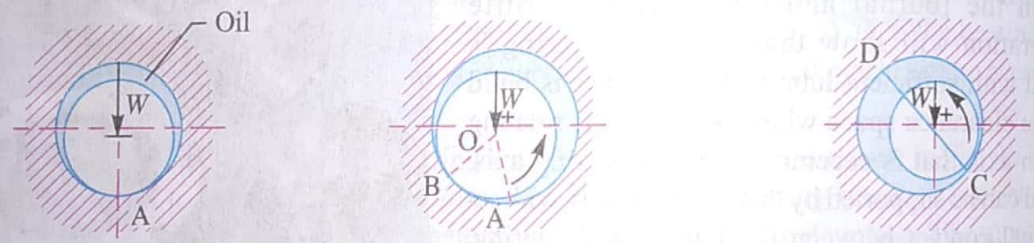
26.6 Important Factors for the Formation of Thick Oil Film in Hydrodynamic Lubricated Bearings

According to Reynolds, the following factors are essential for the formation of a thick film of oil in hydrodynamic lubricated bearings :

1. A continuous supply of oil.
2. A relative motion between the two surfaces in a direction approximately tangential to the surfaces.
3. The ability of one of the surfaces to take up a small inclination to the other surface in the direction of the relative motion.
4. The line of action of resultant oil pressure must coincide with the line of action of the external load between the surfaces.

26.7 Wedge Film Journal Bearings

The load carrying ability of a wedge-film journal bearing results when the journal and/or the bearing rotates relative to the load. The most common case is that of a steady load, a fixed (non-rotating) bearing and a rotating journal. Fig. 26.4 (*a*) shows a journal at rest with metal to metal contact at *A* on the line of action of the supported load. When the journal rotates slowly in the anticlockwise direction, as shown in Fig. 26.4 (*b*), the point of contact will move to *B*, so that the angle *AOB* is the angle of sliding friction of the surfaces in contact at *B*. In the absence of a lubricant, there will be dry metal to metal friction. If a lubricant is present in the clearance space of the bearing and journal, then a thin absorbed film of the lubricant may partly separate the surface, but a continuous fluid film completely separating the surfaces will not exist because of slow speed.



(*a*) At rest. (*b*) Slow speed. (*c*) High speed.

Fig. 26.4. Wedge film journal bearing.

When the speed of the journal is increased, a continuous fluid film is established as in Fig. 26.4 (*c*). The centre of the journal has moved so that the minimum film thickness is at *C*. It may be noted that from *D* to *C* in the direction of motion, the film is continually narrowing and hence is a converging film. The curved converging film may be considered as a wedge shaped film of a slipper bearing wrapped around the journal. A little consideration will show that from *C* to *D* in the direction of rotation, as shown in Fig. 26.4 (*c*), the film is diverging and cannot give rise to a positive pressure or a supporting action.

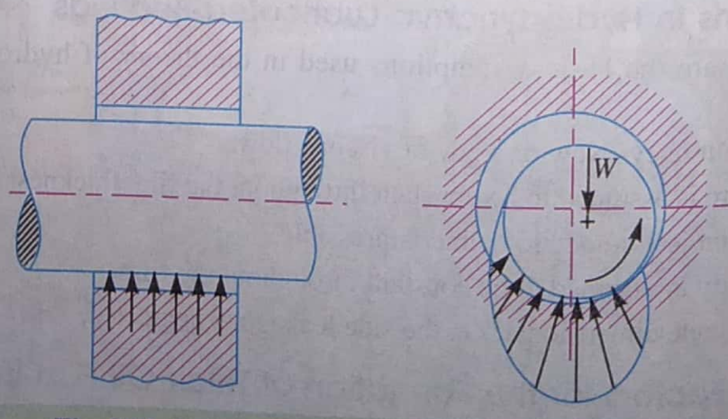


Fig. 26.5. Variation of pressure in the converging film.

6. *Corrosion resistance.* The bearing material should not corrode away under the action of lubricating oil. This property is of particular importance in internal-combustion engines where the same oil is used to lubricate the cylinder walls and bearings. In the cylinder, the lubricating oil comes into contact with hot cylinder walls and may oxidise and collect carbon deposits from the walls.

7. *Thermal conductivity.* The bearing material should be of high thermal conductivity so as to permit the rapid removal of the heat generated by friction.

8. *Thermal expansion.* The bearing material should be of low coefficient of thermal expansion, so that when the bearing operates over a wide range of temperature, there is no undue change in the clearance.

Marine bearings

All these properties as discussed above are, however, difficult to find in any particular bearing material. The various materials are used in practice, depending upon the requirement of the actual service conditions. The choice of material for any application must represent a compromise. The following table shows the comparison of some of the properties of more common metallic bearing materials.

Table 26.1. Properties of metallic bearing materials.

Bearing material	*Fatigue strength*	*Comformability*	*Embeddability*	*Anti scoring*	*Corrosion resistance*	*Thermal conductivity*
Tin base babbit	Poor	Good	Excellent	Excellent	Excellent	Poor
Lead base babbit	Poor to fair	Good	Good	Good to excellent	Fair to good	Poor
Lead bronze	Fair	Poor	Poor	Poor	Good	Fair
Copper lead	Fair	Poor	Poor to fair	Poor to fair	Poor to fair	Fair to good
Aluminium	Good	Poor to fair	Poor	Good	Excellent	Fair
Silver	Excellent	Almost none	Poor	Poor	Excellent	Excellent
Silver lead deposited	Excellent	Excellent	Poor	Fair to good	Excellent	Excellent

26.10 Materials used for Sliding Contact Bearings

The materials commonly used for sliding contact bearings are discussed below :

1. *Babbit metal.* The tin base and lead base babbits are widely used as a bearing material, because they satisfy most requirements for general applications. The babbits are recommended where the maximum bearing pressure (on projected area) is not over 7 to 14 N/mm^2. When applied in automobiles, the babbit is generally used as a thin layer, 0.05 mm to 0.15 mm thick, bonded to an insert or steel shell. The composition of the babbit metals is as follows :

Tin base babbits : Tin 90% ; Copper 4.5% ; Antimony 5% ; Lead 0.5%.

Lead base babbits : Lead 84% ; Tin 6% ; Anitmony 9.5% ; Copper 0.5%.

2. *Bronzes.* The bronzes (alloys of copper, tin and zinc) are generally used in the form of machined bushes pressed into the shell. The bush may be in one or two pieces. The bronzes commonly used for bearing material are gun metal and phosphor bronzes.

The ***gun metal*** (Copper 88% ; Tin 10% ; Zinc 2%) is used for high grade bearings subjected to high pressures (not more than 10 N/mm^2 of projected area) and high speeds.

The ***phosphor bronze*** (Copper 80% ; Tin 10% ; Lead 9% ; Phosphorus 1%) is used for bearings subjected to very high pressures (not more than 14 N/mm^2 of projected area) and speeds.

3. *Cast iron.* The cast iron bearings are usually used with steel journals. Such type of bearings are fairly successful where lubrication is adequate and the pressure is limited to 3.5 N/mm^2 and speed to 40 metres per minute.

4. *Silver.* The silver and silver lead bearings are mostly used in aircraft engines where the fatigue strength is the most important consideration.

5. *Non-metallic bearings.* The various non-metallic bearings are made of carbon-graphite, rubber, wood and plastics. The **carbon-graphite bearings** are self lubricating, dimensionally stable over a wide range of operating conditions, chemically inert and can operate at higher temperatures than other bearings. Such type of bearings are used in food processing and other equipment where contamination by oil or grease must be prohibited. These bearings are also used in applications where the shaft speed is too low to maintain a hydrodynamic oil film.

The ***soft rubber bearings*** are used with water or other low viscosity lubricants, particularly where sand or other large particles are present. In addition to the high degree of embeddability and conformability, the rubber bearings are excellent for absorbing shock loads and vibrations. The rubber bearings are used mainly on marine propeller shafts, hydraulic turbines and pumps.

The ***wood bearings*** are used in many applications where low cost, cleanliness, inattention to lubrication and anti-seizing are important.

Industrial bearings.

The commonly used plastic material for bearings is ***Nylon*** and ***Teflon***. These materials have many characteristics desirable in bearing materials and both can be used dry *i.e.* as a zero film bearing. The Nylon is stronger, harder and more resistant to abrasive wear. It is used for applications in which these properties are important *e.g.* elevator bearings, cams in telephone dials etc. The Teflon is rapidly replacing Nylon as a wear surface or liner for journal and other sliding bearings because of the following properties:

1. It has lower coefficient of friction, about 0.04 (dry) as compared to 0.15 for Nylon.
2. It can be used at higher temperatures up to about 315°C as compared to 120°C for Nylon.
3. It is dimensionally stable because it does not absorb moisture, and
4. It is practically chemically inert.

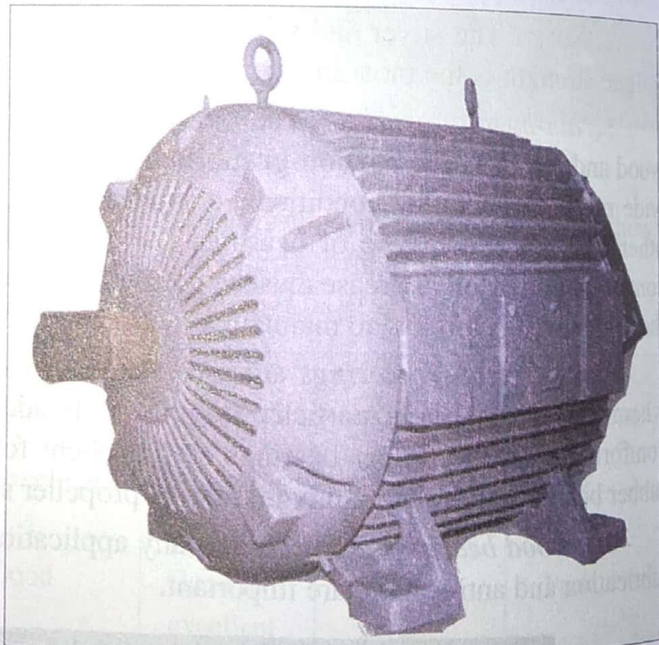
26.11 Lubricants

The lubricants are used in bearings to reduce friction between the rubbing surfaces and to carry away the heat generated by friction. It also protects the bearing against corrosion. All lubricants are classified into the following three groups :

1. Liquid, 2. Semi-liquid, and 3. Solid.

The ***liquid lubricants*** usually used in bearings are mineral oils and synthetic oils. The mineral oils are most commonly used because of their cheapness and stability. The liquid lubricants are usually preferred where they may be retained.

A grease is a ***semi-liquid lubricant*** having higher viscosity than oils. The greases are employed where slow speed and heavy pressure exist and where oil drip from the bearing is undesirable. The ***solid lubricants*** are useful in reducing friction where oil films cannot be maintained because of pressures or temperatures. They should be softer than materials being lubricated. A graphite is the most common of the solid lubricants either alone or mixed with oil or grease.



Wherever moving and rotating parts are present proper lubrication is essential to protect the moving parts from wear and tear and reduce friction.

26.12 Properties of Lubricants

1. ***Viscosity.*** It is the measure of degree of fluidity of a liquid. It is a physical property by virtue of which an oil is able to form, retain and offer resistance to shearing a buffer film-under heat and pressure. The greater the heat and pressure, the greater viscosity is required of a lubricant to prevent thinning and squeezing out of the film.

The fundamental meaning of viscosity may be understood by considering a flat plate moving under a force P parallel to a stationary plate, the two plates being separated by a thin film of a fluid lubricant of thickness h, as shown in Fig. 26.6. The particles of the lubricant adhere strongly to the moving and stationary plates. The motion is accompanied by a linear slip or shear between the particles throughout the entire height (h) of the film thickness. If A is the area of the plate in contact with the lubricant, then the unit shear stress is given by

$$\tau = P / A$$

According to Newton's law of viscous flow, the magnitude of this shear stress varies directly with the velocity gradient (dV / dy). It is assumed that

(a) the lubricant completely fills the space between the two surfaces,

(b) the velocity of the lubricant at each surface is same as that of the surface, and

(c) any flow of the lubricant perpendicular to the velocity of the plate is negligible.

$$\therefore \quad \tau = \frac{P}{A} \propto \frac{dV}{dy} \quad \text{or} \quad \tau = Z \times \frac{dV}{dy}$$

where Z is a constant of proportionality and is known as ***absolute viscosity*** (or simply viscosity) of the lubricant.

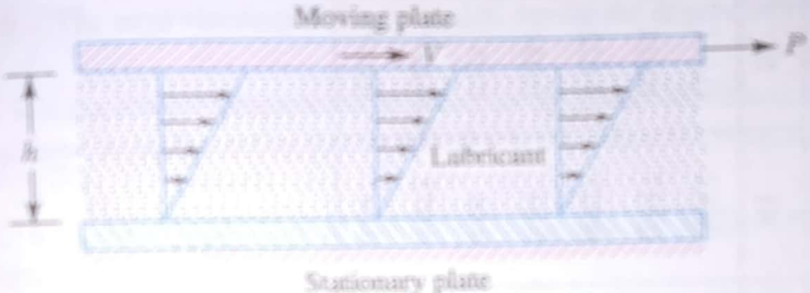


Fig. 26.6. Viscosity.

When the thickness of the fluid lubricant is small which is the case for bearings, then the velocity gradient is very nearly constant as shown in Fig. 26.6, so that

$$\frac{dV}{dy} = \frac{V}{y} = \frac{V}{h}$$

$$\therefore \quad \tau = Z \times \frac{V}{h} \quad \text{or} \quad Z = \tau \times \frac{h}{V}$$

When τ is in N/m², h is in metres and V is in m/s, then the unit of absolute viscosity is given by

$$Z = \tau \times \frac{h}{V} = \frac{\text{N}}{\text{m}^2} \times \frac{\text{m}}{\text{m/s}} = \text{N-s/m}^2$$

However, the common practice is to express the absolute viscosity in mass units, such that

$$1 \text{ N-s/m}^2 = \frac{1 \text{kg-m}}{s^2} \times \frac{s}{\text{m}^2} = 1 \text{ kg/m-s} \qquad \ldots (\because 1 \text{ N} = 1 \text{ kg-m}/s^2)$$

Thus the unit of absolute viscosity in S.I. units is kg / m-s.

The viscocity of the lubricant is measured by Saybolt universal viscometer. It determines the time required for a standard volume of oil at a certain temperature to flow under a certain head through a tube of standard diameter and length. The time so determined in seconds is the Saybolt universal viscosity. In order to convert Saybolt universal viscosity in seconds to absolute viscosity (in kg / m-s), the following formula may be used:

$$Z = \text{Sp. gr. of oil} \left(0.000\,22\,S - \frac{0.18}{S} \right) \text{kg/m-s} \qquad \ldots (i)$$

where

Z = Absolute viscosity at temperature t in kg / m-s, and

S = Saybolt universal viscosity in seconds.

The variation of absolute viscosity with temperature for commonly used lubricating oils is shown in Table 26.2 on the next page.

2. Oiliness. It is a joint property of the lubricant and the bearing surfaces in contact. It is a measure of the lubricating qualities under boundary conditions where base metal to metal is prevented only by absorbed film. There is no absolute measure of oiliness.

3. *Density.* This property has no relation to lubricating value but is useful in changing the kinematic viscosity to absolute viscosity. Mathematically

$$\text{Absolute viscosity} = \rho \times \text{Kinematic viscosity (in m}^2\text{/s)}$$

where ρ = Density of the lubricating oil.

The density of most of the oils at 15.5°C varies from 860 to 950 kg / m³ (the average value may be taken as 900 kg / m³). The density at any other temperature (t) may be obtained from the following relation, *i.e.*

$$\rho_t = \rho_{15.5} - 0.000\ 657\ t$$

where $\rho_{15.5}$ = Density of oil at 15.5° C.

4. *Viscosity index.* The term viscosity index is used to denote the degree of variation of viscosity with temperature.

5. *Flash point.* It is the lowest temperature at which an oil gives off sufficient vapour to support a momentary flash without actually setting fire to the oil when a flame is brought within 6 mm at the surface of the oil.

6. *Fire point.* It is the temperature at which an oil gives off sufficient vapour to burn it continuously when ignited.

7. *Pour point or freezing point.* It is the temperature at which an oil will cease to flow when cooled.

26.13 Terms used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in Fig. 26.7, in which O is the centre of the journal and O' is the centre of the bearing.

Let D = Diameter of the bearing,

d = Diameter of the journal,

and l = Length of the bearing.

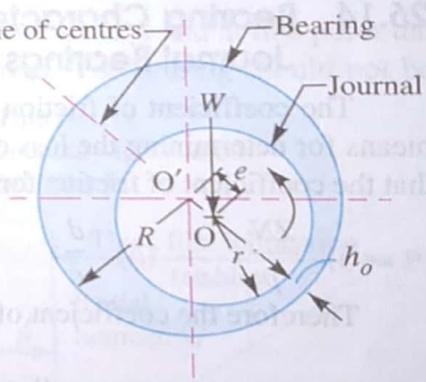


Fig. 26.7. Hydrodynamic journal bearing.

The following terms used in hydrodynamic journal bearing are important from the subject point of view :

1. *Diametral clearance.* It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance,

$$c = D - d$$

Note : The diametral clearance (c) in a bearing should be small enough to produce the necessary velocity gradient, so that the pressure built up will support the load. Also the small clearance has the advantage of decreasing side leakage. However, the allowance must be made for manufacturing tolerances in the journal and bushing. A commonly used clearance in industrial machines is 0.025 mm per cm of journal diameter.

2. *Radial clearance.* It is the difference between the radii of the bearing and the journal. Mathematically, radial clearance,

$$c_1 = R - r = \frac{D - d}{2} = \frac{c}{2}$$

3. *Diametral clearance ratio.* It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio

$$= \frac{c}{d} = \frac{D - d}{d}$$

4. ***Eccentricity.*** It is the radial distance between the centre (O) of the bearing and the displaced centre (O') of the bearing under load. It is denoted by e.

5. ***Minimum oil film thickness.*** It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h_0 and occurs at the line of centres as shown in Fig. 26.7. Its value may be assumed as $c / 4$.

6. ***Attitude or eccentricity ratio.*** It is the ratio of the eccentricity to the radial clearance. Mathematically, attitude or eccentricity ratio,

$$\varepsilon = \frac{e}{c_1} = \frac{c_1 - h_0}{c_1} = 1 - \frac{h_0}{c_1} = 1 - \frac{2h_0}{c} \qquad \dots (\because c_1 = c/2)$$

7. ***Short and long bearing.*** If the ratio of the length to the diameter of the journal (*i.e.* l / d) is less than 1, then the bearing is said to be **short bearing.** On the other hand, if l / d is greater than 1, then the bearing is known as ***long bearing.***

Notes : 1. When the length of the journal (l) is equal to the diameter of the journal (d), then the bearing is called ***square bearing.***

2. Because of the side leakage of the lubricant from the bearing, the pressure in the film is atmospheric at the ends of the bearing. The average pressure will be higher for a long bearing than for a short or square bearing. Therefore, from the stand point of side leakage, a bearing with a large l / d ratio is preferable. However, space requirements, manufacturing, tolerances and shaft deflections are better met with a short bearing. The value of l / d may be taken as 1 to 2 for general industrial machinery. In crank shaft bearings, the l / d ratio is frequently less than 1.

Axle bearings

26.14 Bearing Characteristic Number and Bearing Modulus for Journal Bearings

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, *i.e.*

(*i*) $\dfrac{ZN}{p}$; (*ii*) $\dfrac{d}{c}$; and (*iii*) $\dfrac{l}{d}$

Therefore the coefficient of friction may be expressed as

$$\mu = \phi\left(\frac{ZN}{p}, \frac{d}{c}, \frac{l}{d}\right)$$

where
μ = Coefficient of friction,
ϕ = A functional relationship,
Z = Absolute viscosity of the lubricant, in kg / m-s,
N = Speed of the journal in r.p.m.,
p = Bearing pressure on the projected bearing area in N/mm^2,
= Load on the journal $\div$ $l \times d$
d = Diameter of the journal,
l = Length of the bearing, and
c = Diametral clearance.

The factor ZN / p is termed as ***bearing characteristic number*** and is a dimensionless number. The variation of coefficient of friction with the operating values of bearing characteristic number (ZN / p) as obtained by McKee brothers (S.A. McKee and T.R. McKee) in an actual test of friction is shown in Fig. 26.8. The factor ZN/p helps to predict the performance of a bearing.

The part of the curve PQ represents the region of thick film lubrication. Between Q and R, the viscosity (Z) or the speed (N) are so low, or the pressure (p) is so great that their combination ZN/p will reduce the film thickness so that partial metal to metal contact will result. The thin film or boundary lubrication or imperfect lubrication exists between R and S on the curve. This is the region where the viscosity of the lubricant ceases to be a measure of friction characteristics but the oiliness of the lubricant is effective in preventing complete metal to metal contact and seizure of the parts.

Clutch bearing

It may be noted that the part PQ of the curve represents stable operating conditions, since from any point of stability, a decrease in viscosity (Z) will reduce ZN/p. This will result in a decrease in coefficient of friction (μ) followed by a lowering of bearing temperature that will raise the viscosity (Z).

From Fig. 26.8, we see that the minimum amount of friction occurs at A and at this point the value of ZN/p is known as ***bearing modulus*** which is denoted by K. The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. This will result in high friction, wear and heating. In order to prevent such conditions, the bearing should be designed for a value of ZN/p at least three times the minimum value of bearing modulus (K). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of $ZN/p = 15\,K$ may be used.

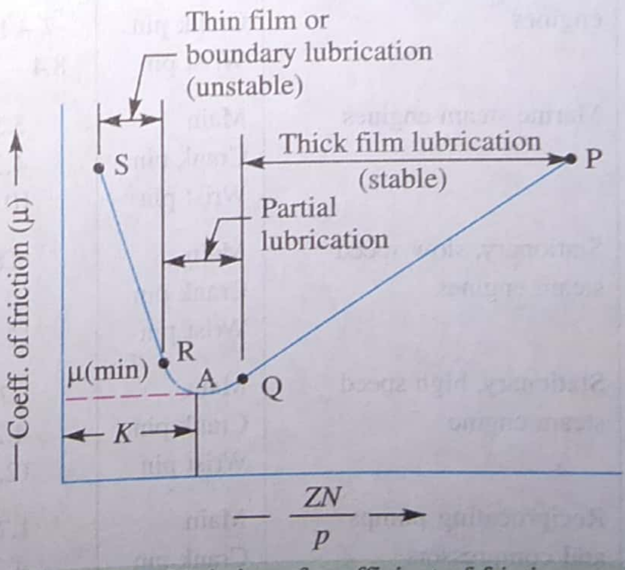


Fig. 26.8. Variation of coefficient of friction with ZN/p.

From above, it is concluded that when the value of ZN/p is greater than K, then the bearing will operate with thick film lubrication or under hydrodynamic conditions. On the other hand, when the value of ZN/p is less than K, then the oil film will rupture and there is a metal to metal contact.

26.15 Coefficient of Friction for Journal Bearings

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used.

*Coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k \quad \text{... (when } Z \text{ is in kg / m-s and } p \text{ is in N / mm}^2\text{)}$$

where *Z, N, p, d* and *c* have usual meanings as discussed in previous article, and

k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (*i.e.* l / d).

= 0.002 for l / d ratios of 0.75 to 2.8.

The operating values of ZN / p should be compared with values given in Table 26.3 to ensure safe margin between operating conditions and the point of film breakdown.

Table 26.3. Design values for journal bearings.

Machinery	Bearing	Maximum bearing pressure (p) in N/mm²	Operating values			
			Absolute Viscosity (Z) in kg/m-s	ZN/p Z in kg/m-s p in N/mm²	$\frac{c}{d}$	$\frac{l}{d}$
Automobile and air-craft engines	Main	5.6 – 12	0.007	2.1	—	0.8 – 1.8
	Crank pin	10.5 – 24.5	0.008	1.4		0.7 – 1.4
	Wrist pin	16 – 35	0.008	1.12		1.5 – 2.2
Four stroke-Gas and oil engines	Main	5 – 8.5	0.02	2.8	0.001	0.6 – 2
	Crank pin	9.8 – 12.6	0.04	1.4		0.6 – 1.5
	Wrist pin	12.6 – 15.4	0.065	0.7		1.5 – 2
Two stroke-Gas and oil engines	Main	3.5 – 5.6	0.02	3.5	0.001	0.6 – 2
	Crank pin	7 – 10.5	0.04	1.8		0.6 – 1.5
	Wrist pin	8.4 – 12.6	0.065	1.4		1.5 – 2
Marine steam engines	Main	3.5	0.03	2.8	0.001	0.7 – 1.5
	Crank pin	4.2	0.04	2.1		0.7 – 1.2
	Wrist pin	10.5	0.05	1.4		1.2 – 1.7
Stationary, slow speed steam engines	Main	2.8	0.06	2.8	0.001	1 – 2
	Crank pin	10.5	0.08	0.84		0.9 – 1.3
	Wrist pin	12.6	0.06	0.7		1.2 – 1.5
Stationary, high speed steam engine	Main	1.75	0.015	3.5	0.001	1.5 – 3
	Crank pin	4.2	0.030	0.84		0.9 – 1.5
	Wrist pin	12.6	0.025	0.7		13 – 1.7
Reciprocating pumps and compressors	Main	1.75	0.03	4.2	0.001	1 – 2.2
	Crank pin	4.2	0.05	2.8		0.9 – 1.7
	Wrist pin	7.0	0.08	1.4		1.5 – 2.0
Steam locomotives	Driving axle	3.85	0.10	4.2	0.001	1.6 – 1.8
	Crank pin	14	0.04	0.7		0.7 – 1.1
	Wrist pin	28	0.03	0.7		0.8 – 1.3

* This is the equation of a straight line portion in the region of thick film lubrication (*i.e.* line *PQ*) as shown in Fig. 26.8.

Machinery	Bearing	Maximum bearing pressure (p) in N/mm²	Operating values			
			Absolute Viscosity (Z) in kg/m-s	ZN/p Z in kg/m-s p in N/mm²	$\frac{c}{d}$	$\frac{l}{d}$
Railway cars	Axle	3.5	0.1	7	0.001	1.8 – 2
Steam turbines	Main	0.7 – 2	0.002 – 0.016	14	0.001	1 – 2
Generators, motors, centrifugal pumps	Rotor	0.7 – 1.4	0.025	28	0.0013	1 – 2
Transmission shafts	Light, fixed	0.175	0.025-	7	0.001	2 – 3
	Self -aligning	1.05	0.060	2.1		2.5 – 4
	Heavy	1.05		2.1		2– 3
Machine tools	Main	2.1	0.04	0.14	0.001	1–4
Punching and shearing machines	Main	28	0.10	—	0.001	1–2
	Crank pin	56				
Rolling Mills	Main	21	0.05	1.4	0.0015	1–1.5

26.16 Critical Pressure of the Journal Bearing

The pressure at which the oil film breaks down so that metal to metal contact begins, is known as **critical pressure** or the **minimum operating pressure** of the bearing. It may be obtained by the following empirical relation, *i.e.*

Critical pressure or minimum operating pressure,

$$p = \frac{ZN}{4.75 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{d+l}\right) \text{ N/mm}^2 \quad \text{...(when Z is in kg / m-s)}$$

26.17 Sommerfeld Number

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically,

$$\text{Sommerfeld number} = \frac{ZN}{p}\left(\frac{d}{c}\right)^2$$

For design purposes, its value is taken as follows :

$$\frac{ZN}{p}\left(\frac{d}{c}\right)^2 = 14.3 \times 10^6 \quad \text{... (when } Z \text{ is in kg / m-s and } p \text{ is in N / mm}^2\text{)}$$

26.18 Heat Generated in a Journal Bearing

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$$Q_g = \mu . W.V \text{ N-m/s or J/s or watts} \quad ...(i)$$

where μ = Coefficient of friction,

W = Load on the bearing in N,

= Pressure on the bearing in N/mm² × Projected area of the bearing in mm² = $p\,(l \times d)$,

V = Rubbing velocity in m/s $= \dfrac{\pi d.N}{60}$, d is in metres, and

N = Speed of the journal in r.p.m.

After the thermal equilibrium has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing,

$$Q_d = C.A\,(t_b - t_a) \text{ J/s or W} \qquad \ldots (\because 1 \text{ J/s} = 1 \text{ W}) \quad \ldots(ii)$$

where C = Heat dissipation coefficient in W/m²/°C,

A = Projected area of the bearing in m² $= l \times d$,

t_b = Temperature of the bearing surface in °C, and

t_a = Temperature of the surrounding air in °C.

The value of C have been determined experimentally by O. Lasche. The values depend upon the type of bearing, its ventilation and the temperature difference. The average values of C (in W/m²/°C), for journal bearings may be taken as follows :

For unventilated bearings (Still air)

= 140 to 420 W/m²/°C

For well ventilated bearings

= 490 to 1400 W/m²/°C

It has been shown by experiments that the temperature of the bearing (t_b) is approximately mid-way between the temperature of the oil film (t_0) and the temperature of the outside air (t_a). In other words,

$$t_b - t_a = \frac{1}{2}\,(t_0 - t_a)$$

Notes : 1. For well designed bearing, the temperature of the oil film should not be more than 60°C, otherwise the viscosity of the oil decreases rapidly and the operation of the bearing is found to suffer. The temperature of the oil film is often called as the ***operating temperature*** of the bearing.

2. In case the temperature of the oil film is higher, then the bearing is cooled by circulating water through coils built in the bearing.

3. The mass of the oil to remove the heat generated at the bearing may be obtained by equating the heat generated to the heat taken away by the oil. We know that the heat taken away by the oil,

$$Q_t = m.S.t \text{ J/s or watts}$$

where m = Mass of the oil in kg / s,

S = Specific heat of the oil. Its value may be taken as 1840 to 2100 J / kg / °C,

t = Difference between outlet and inlet temperature of the oil in °C.

26.19 Design Procedure for Journal Bearing

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known.

1. Determine the bearing length by choosing a ratio of l/d from Table 26.3.
2. Check the bearing pressure, $p = W / l.d$ from Table 26.3 for probable satisfactory value.
3. Assume a lubricant from Table 26.2 and its operating temperature (t_0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations such as steam turbines.
4. Determine the operating value of ZN/p for the assumed bearing temperature and check this value with corresponding values in Table 26.3, to determine the possibility of maintaining fluid film operation.
5. Assume a clearance ratio c/d from Table 26.3.
6. Determine the coefficient of friction (μ) by using the relation as discussed in Art. 26.15.

Journal bearings are used in helicopters, primarily in the main rotor axis and in the landing gear for fixed wing aircraft.

7. Determine the heat generated by using the relation as discussed in Art. 26.18.
8. Determine the heat dissipated by using the relation as discussed in Art. 26.18.
9. Determine the thermal equilibrium to see that the heat dissipated becomes atleast equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

Example 26.1. *Design a journal bearing for a centrifugal pump from the following data :*

Load on the journal = 20 000 N; Speed of the journal = 900 r.p.m.; Type of oil is SAE 10, for which the absolute viscosity at 55°C = 0.017 kg / m-s; Ambient temperature of oil = 15.5°C ; Maximum bearing pressure for the pump = 1.5 N / mm².

Calculate also mass of the lubricating oil required for artificial cooling, if rise of temperature of oil be limited to 10°C. Heat dissipation coefficient = 1232 W/m²/°C.

Solution. Given : $W = 20\ 000$ N ; $N = 900$ r.p.m. ; $t_0 = 55°C$; $Z = 0.017$ kg/m-s ; $t_a = 15.5°C$; $p = 1.5$ N/mm² ; $t = 10°C$; $C = 1232$ W/m²/°C

The journal bearing is designed as discussed in the following steps :

1. First of all, let us find the length of the journal (l). Assume the diameter of the journal (d) as 100 mm. From Table 26.3, we find that the ratio of l/d for centrifugal pumps varies from 1 to 2. Let us take $l/d = 1.6$.

$$\therefore \quad l = 1.6\,d = 1.6 \times 100 = 160 \text{ mm} \quad \textbf{Ans.}$$

2. We know that bearing pressure,

$$p = \frac{W}{l.d} = \frac{20\ 000}{160 \times 100} = 1.25$$

Since the given bearing pressure for the pump is 1.5 N/mm², therefore the above value of p is safe and hence the dimensions of l and d are safe.

3. $$\frac{Z.N}{p} = \frac{0.017 \times 900}{1.25} = 12.24$$

From Table 26.3, we find that the operating value of

$$\frac{Z.N}{p} = 28$$

We have discussed in Art. 26.14, that the minimum value of the bearing modulus at which the oil film will break is given by

$$3\,K = \frac{ZN}{p}$$

∴ Bearing modulus at the minimum point of friction,

$$K = \frac{1}{3}\left(\frac{Z.N}{p}\right) = \frac{1}{3} \times 28 = 9.33$$

Since the calculated value of bearing characteristic number $\left(\frac{Z.N}{p} = 12.24\right)$ is more than 9.33, therefore the bearing will operate under hydrodynamic conditions.

4. From Table 26.3, we find that for centrifugal pumps, the clearance ratio (c/d)

$$= 0.0013$$

5. We know that coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8} \times 12.24 \times \frac{1}{0.0013} + 0.002$$

$$= 0.0031 + 0.002 = 0.0051 \qquad \text{... [From Art. 26.13, } k = 0.002]$$

6. Heat generated,

$$Q_g = \mu\, W\, V = \mu\, W\left(\frac{\pi\, d.N}{60}\right) \text{W} \qquad \text{...}\left(\because V = \frac{\pi d.N}{60}\right)$$

$$= 0.0051 \times 20\,000\left(\frac{\pi \times 0.1 \times 900}{60}\right) = 480.7 \text{ W}$$

... (d is taken in metres)

7. Heat dissipated,

$$Q_d = C.A\,(t_b - t_a) = C.l.d\,(t_b - t_a) \text{ W} \qquad \text{...}(\because A = l \times d)$$

We know that

$$(t_b - t_a) = \tfrac{1}{2}\,(t_0 - t_a) = \tfrac{1}{2}\,(55° - 15.5°) = 19.75°\text{C}$$

$$\therefore \quad Q_d = 1232 \times 0.16 \times 0.1 \times 19.75 = 389.3 \text{ W}$$

... (l and d are taken in metres)

We see that the heat generated is greater than the heat dissipated which indicates that the bearing is warming up. Therefore, either the bearing should be redesigned by taking $t_0 = 63$°C or the bearing should be cooled artificially.

We know that the amount of artificial cooling required

$$= \text{Heat generated} - \text{Heat dissipated} = Q_g - Q_d$$

$$= 480.7 - 389.3 = 91.4 \text{ W}$$

Mass of lubricating oil required for artificial cooling

Let m = Mass of the lubricating oil required for artificial cooling in kg / s.

We know that the heat taken away by the oil,

$$Q_t = m.S.t = m \times 1900 \times 10 = 19\,000\, m \text{ W}$$

... [∵ Specific heat of oil (S) = 1840 to 2100 J/kg/°C]

Equating this to the amount of artificial cooling required, we have

$$19\,000\, m = 91.4$$

$$\therefore \quad m = 91.4 / 19\,000 = 0.0048 \text{ kg / s} = 0.288 \text{ kg / min} \quad \textbf{Ans.}$$

Example 26.2. *The load on the journal bearing is 150 kN due to turbine shaft of 300 mm diameter running at 1800 r.p.m. Determine the following :*

1. Length of the bearing if the allowable bearing pressure is 1.6 N/mm², and 2. Amount of heat to be removed by the lubricant per minute if the bearing temperature is 60°C and viscosity of the oil at 60°C is 0.02 kg/m-s and the bearing clearance is 0.25 mm.

Axle bearing

Solution. Given : $W = 150$ kN $= 150 \times 10^3$ N ; $d = 300$ mm $= 0.3$ m ; $N = 1800$ r.p.m. ; $p = 1.6$ N/mm² ; $Z = 0.02$ kg / m-s ; $c = 0.25$ mm

1. Length of the bearing

Let l = Length of the bearing in mm.

We know that projected bearing area,

$$A = l \times d = l \times 300 = 300\, l \text{ mm}^2$$

and allowable bearing pressure (p),

$$1.6 = \frac{W}{A} = \frac{150 \times 10^3}{300\, l} = \frac{500}{l}$$

$\therefore$ $l = 500 / 1.6 = 312.5$ mm **Ans.**

2. Amount of heat to be removed by the lubricant

We know that coefficient of friction for the bearing,

$$\mu = \frac{33}{10^8}\left(\frac{Z.N}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8}\left(\frac{0.02 \times 1800}{1.6}\right)\left(\frac{300}{0.25}\right) + 0.002$$

$$= 0.009 + 0.002 = 0.011$$

Rubbing velocity,

$$V = \frac{\pi\, d.N}{60} = \frac{\pi \times 0.3 \times 1800}{60} = 28.3 \text{ m/s}$$

$\therefore$ Amount of heat to be removed by the lubricant,

$$Q_g = \mu.W.V = 0.011 \times 150 \times 10^3 \times 28.3 = 46\,695 \text{ J/s or W}$$

... (1 J/s = 1 W)

$= 46.695$ kW **Ans.**

Example 26.3. *A full journal bearing of 50 mm diameter and 100 mm long has a bearing pressure of 1.4 N/mm². The speed of the journal is 900 r.p.m. and the ratio of journal diameter to the diametral clearance is 1000. The bearing is lubricated with oil whose absolute viscosity at the operating temperature of 75°C may be taken as 0.011 kg/m-s. The room temperature is 35°C. Find : 1. The amount of artificial cooling required, and 2. The mass of the lubricating oil required, if the difference between the outlet and inlet temperature of the oil is 10°C. Take specific heat of the oil as 1850 J / kg / °C.*

Solution. Given : $d = 50$ mm $= 0.05$ m ; $l = 100$ mm $= 0.1$ m; $p = 1.4$ N/mm² ; $N = 900$ r.p.m. ; $d / c = 1000$; $Z = 0.011$ kg / m-s ; $t_0 = 75$°C ; $t_a = 35$°C ; $t = 10$°C ; $S = 1850$ J/kg / °C

1. Amount of artificial cooling required

We know that the coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8}\left(\frac{0.011 \times 900}{1.4}\right)(1000) + 0.002$$

$$= 0.002\,33 + 0.002 = 0.004\,33$$

Load on the bearing,

$$W = p \times d.l = 1.4 \times 50 \times 100 = 7000 \text{ N}$$

CHAPTER

27

Rolling Contact Bearings

27.1 Introduction

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction. Due to this low friction offered by rolling contact bearings, these are called ***antifriction bearings***.

27.2 Advantages and Disadvantages of Rolling Contact Bearings Over Sliding Contact Bearings

The following are some advantages and disadvantages of rolling contact bearings over sliding contact bearings.

Advantages

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

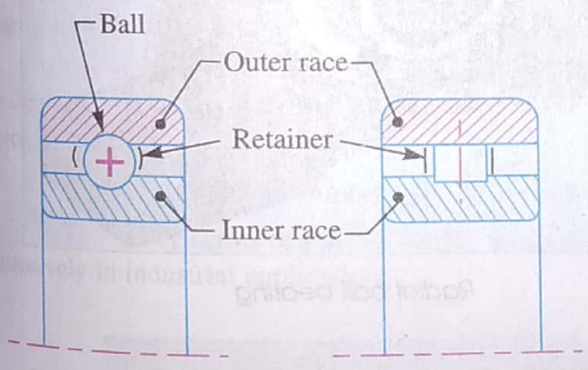
Disadvantages

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.

27.3 Types of Rolling Contact Bearings

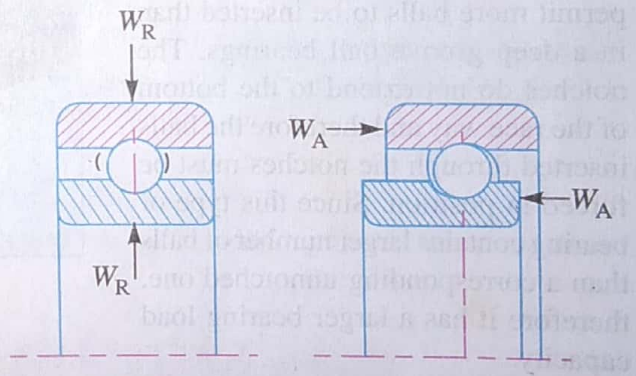
Following are the two types of rolling contact bearings:

1. Ball bearings; and 2. Roller bearings.



(*a*) Ball bearing. (*b*) Roller bearing.

Fig. 27.1. Ball and roller bearings.



(*a*) Radial ball bearing. (*b*) Thrust ball bearing.

Fig. 27.2. Radial and thrust ball bearings.

The ***ball and roller bearings*** consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig. 27.1. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads.

The rolling contact bearings, depending upon the load to be carried, are classified as :

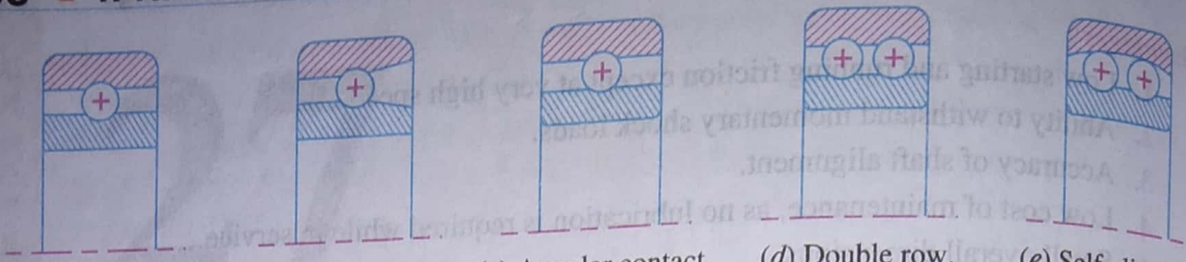
(*a*) Radial bearings, and (*b*) Thrust bearings.

The radial and thrust ball bearings are shown in Fig. 27.2 (*a*) and (*b*) respectively. When a ball bearing supports only a radial load (W_R), the plane of rotation of the ball is normal to the centre line of the bearing, as shown in Fig. 27.2 (*a*). The action of thrust load (W_A) is to shift the plane of rotation of the balls, as shown in Fig. 27.2 (*b*). The radial and thrust loads both may be carried simultaneously.

27.4 Types of Radial Ball Bearings

Following are the various types of radial ball bearings:

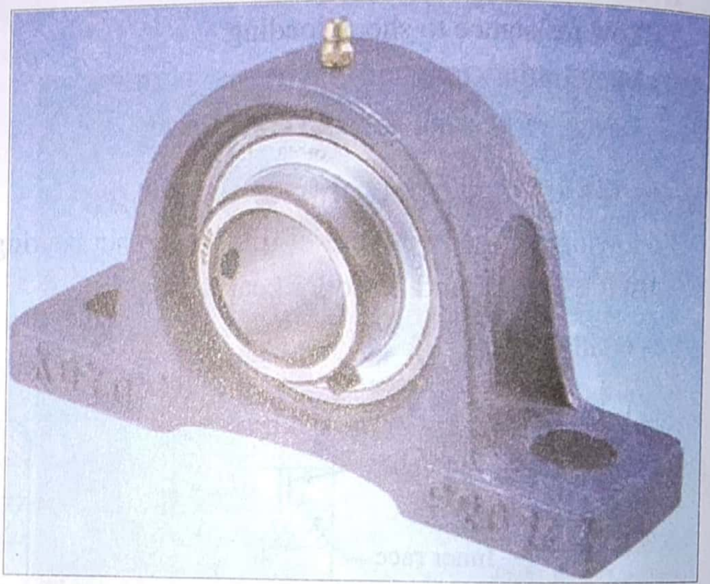
1. ***Single row deep groove bearing.*** A single row deep groove bearing is shown in Fig. 27.3 (*a*).



(a) Single row deep groove. (b) Filling notch. (c) Angular contact. (d) Double row. (e) Self-aligning.

Fig. 27.3. Types of radial ball bearings.

During assembly of this bearing, the races are offset and the maximum number of balls are placed between the races. The races are then centred and the balls are symmetrically located by the use of a retainer or cage. The deep groove ball bearings are used due to their high load carrying capacity and suitability for high running speeds. The load carrying capacity of a ball bearing is related to the size and number of the balls.



Radial ball bearing

2. *Filling notch bearing.* A filling notch bearing is shown in Fig. 27.3 (*b*). These bearings have notches in the inner and outer races which permit more balls to be inserted than in a deep groove ball bearings. The notches do not extend to the bottom of the race way and therefore the balls inserted through the notches must be forced in position. Since this type of bearing contains larger number of balls than a corresponding unnotched one, therefore it has a larger bearing load capacity.

3. *Angular contact bearing.* An angular contact bearing is shown in Fig. 27.3 (*c*). These bearings have one side of the outer race cut away to permit the insertion of more balls than in a deep groove bearing but without having a notch cut into both races. This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load. The angular contact bearings are usually used in pairs so that thrust loads may be carried in either direction.

4. *Double row bearing.* A double row bearing is shown in Fig. 27.3 (*d*). These bearings may be made with radial or angular contact between the balls and races. The double row bearing is appreciably narrower than two single row bearings. The load capacity of such bearings is slightly less than twice that of a single row bearing.

5. *Self-aligning bearing.* A self-aligning bearing is shown in Fig. 27.3 (*e*). These bearings permit shaft deflections within 2-3 degrees. It may be noted that normal clearance in a ball bearing are too small to accommodate any appreciable misalignment of the shaft relative to the housing. If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur. Following are the two types of self-aligning bearings :

(*a*) Externally self-aligning bearing, and (*b*) Internally self-aligning bearing.

In an ***externally self-aligning bearing,*** the outside diameter of the outer race is ground to a spherical surface which fits in a mating spherical surface in a housing, as shown in Fig. 27.3 (*e*). In case of ***internally self-aligning bearing,*** the inner surface of the outer race is ground to a spherical surface. Consequently, the outer race may be displaced through a small angle without interfering with the normal operation of the bearing. The internally self-aligning ball bearing is interchangeable with other ball bearings.

27.5 Standard Dimensions and Designations of Ball Bearings

The dimensions that have been standardised on an international basis are shown in Fig. 27.4. These dimensions are a function of the bearing bore and the series of bearing. The standard dimensions are given in millimetres. There is no standard for the size and number of steel balls.

The bearings are designated by a number. In general, the number consists of atleast three digits. Additional digits or letters are used to indicate special features *e.g.* deep groove, filling notch etc. The last three digits give the series and the bore of the bearing. The last two digits from 04 onwards, when multiplied by 5, give the bore diameter in millimetres. The third from the last digit designates the series of the bearing. The most common ball bearings are available in four series as follows :

1. Extra light (100),
2. Light (200),
3. Medium (300),
4. Heavy (400)

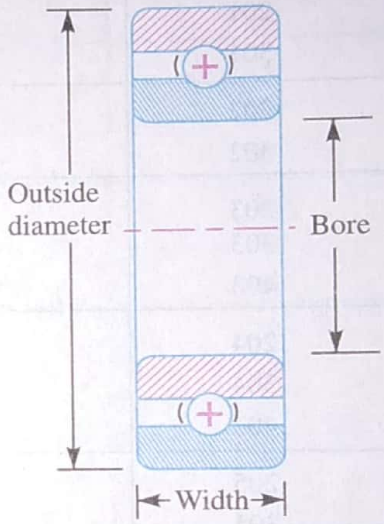


Fig. 27.4. Standard designations of ball bearings.

Notes : 1. If a bearing is designated by the number 305, it means that the bearing is of medium series whose bore is 05 × 5, *i.e.*, 25 mm.

2. The extra light and light series are used where the loads are moderate and shaft sizes are comparatively large and also where available space is limited.

3. The medium series has a capacity 30 to 40 per cent over the light series.

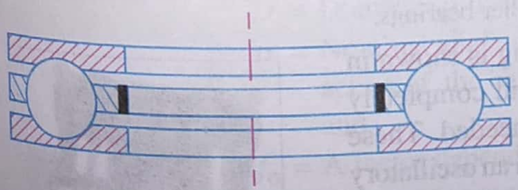
4. The heavy series has 20 to 30 per cent capacity over the medium series. This series is not used extensively in industrial applications.

Oilless bearings made using powder metallergy.

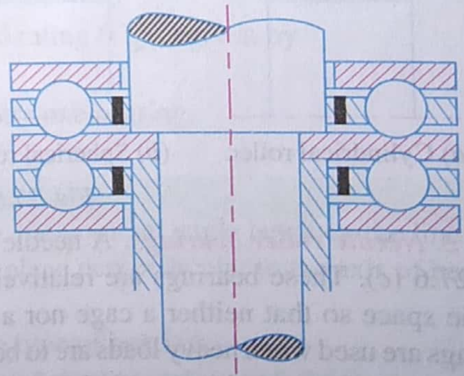
Bearing No.	Bore (mm)	Outside diameter	Width (mm)
213	65	120	23
313		140	33
413		160	37
214	70	125	24
314		150	35
414		180	42
215	75	130	25
315		160	37
415		190	45
216	80	140	26
316		170	39
416		200	48
217	85	150	28
317		180	41
417		210	52
218	90	160	30
318		190	43
418		225	54

27.6 Thrust Ball Bearings

The thrust ball bearings are used for carrying thrust loads exclusively and at speeds below 2000 r.p.m. At high speeds, centrifugal force causes the balls to be forced out of the races. Therefore at high speeds, it is recommended that angular contact ball bearings should be used in place of thrust ball bearings.



(a) Single direction thrust ball bearing.



(b) Double direction thrust ball bearing.

Fig. 27.5. Thrust ball bearing.

A thrust ball bearing may be a single direction, flat face as shown in Fig. 27.5 (a) or a double direction with flat face as shown in Fig. 27.5 (b).

27.7 Types of Roller Bearings

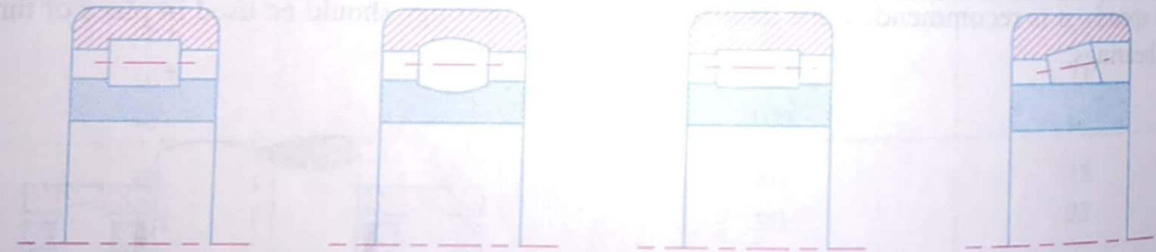
Following are the principal types of roller bearings :

1. *Cylindrical roller bearings.* A cylindrical roller bearing is shown in Fig. 27.6 (a). These bearings have short rollers guided in a cage. These bearings are relatively rigid against radial motion

and have the lowest coefficient of friction of any form of heavy duty rolling-contact bearings. Such type of bearings are used in high speed service.

Radial ball bearing

2. *Spherical roller bearings.* A spherical roller bearing is shown in Fig. 27.6 (*b*). These bearings are self-aligning bearings. The self-aligning feature is achieved by grinding one of the races in the form of sphere. These bearings can normally tolerate angular misalignment in the order of $\pm 1\frac{1}{2}^\circ$ and when used with a double row of rollers, these can carry thrust loads in either direction.



(*a*) Cylindrical roller. (*b*) Spherical roller. (*c*) Needle roller. (*d*) Tapered roller.

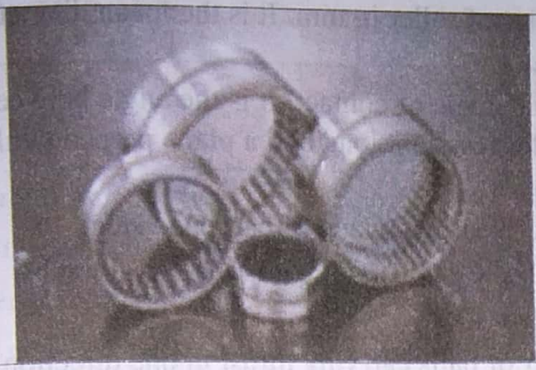
Fig. 27.6. Types of roller bearings.

3. *Needle roller bearings.* A needle roller bearing is shown in Fig. 27.6 (*c*). These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed. These bearings are used when heavy loads are to be carried with an oscillatory motion, *e.g.* piston pin bearings in heavy duty diesel engines, where the reversal of motion tends to keep the rollers in correct alignment.

Cylindrical roller bearings

4. *Tapered roller bearings.* A tapered roller bearing is shown in Fig. 27.6 (*d*). The rollers and race ways of these bearings are truncated cones whose elements intersect at a common point. Such type of bearings can carry both radial and thrust loads. These bearings are available in various combinations as double row bearings and with different cone angles for use with different relative magnitudes of radial and thrust loads.

Spherical roller bearings



Needle roller bearings

Tapered roller bearings

27.8 Basic Static Load Rating of Rolling Contact Bearings

The load carried by a non-rotating bearing is called a static load. The ***basic static load rating*** is defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which corresponds to a total permanent deformation of the ball (or roller) and race, at the most heavily stressed contact, equal to 0.0001 times the ball (or roller) diameter.

In single row angular contact ball bearings, the basic static load relates to the radial component of the load, which causes a purely radial displacement of the bearing rings in relation to each other.

Note : The permanent deformation which appear in balls (or rollers) and race ways under static loads of moderate magnitude, increase gradually with increasing load. The permissible static load is, therefore, dependent upon the permissible magnitude of permanent deformation. Experience shows that a total permanent deformation of 0.0001 times the ball (or roller) diameter, occurring at the most heavily loaded ball (or roller) and race contact can be tolerated in most bearing applications without impairment of bearing operation.

In certain applications where subsequent rotation of the bearing is slow and where smoothness and friction requirements are not too exacting, a much greater total permanent deformation can be permitted. On the other hand, where extreme smoothness is required or friction requirements are critical, less total permanent deformation may be permitted.

According to IS : 3823–1984, the basic static load rating (C_0) in newtons for ball and roller bearings may be obtained as discussed below :

1. For radial ball bearings, the basic static radial load rating (C_0) is given by

$$C_0 = f_0.i.Z.D^2 \cos \alpha$$

where
i = Number of rows of balls in any one bearing,
Z = Number of ball per row,
D = Diameter of balls, in mm,
α = Nominal angle of contact *i.e.* the nominal angle between the line of action of the ball load and a plane perpendicular to the axis of bearing, and
f_0 = A factor depending upon the type of bearing.

The value of factor (f_0) for bearings made of hardened steel are taken as follows :

f_0 = 3.33, for self-aligning ball bearings
= 12.3, for radial contact and angular contact groove ball bearings.

2. For radial roller bearings, the basic static radial load rating is given by

$$C_0 = f_0.i.Z.l_e.D \cos \alpha$$

where
i = Number of rows of rollers in the bearing,
Z = Number of rollers per row,
l_e = Effective length of contact between one roller and that ring (or washer) where the contact is the shortest (in mm). It is equal to the overall length of roller *minus* roller chamfers or grinding undercuts,

D = Diameter of roller in mm. It is the mean diameter in case of tapered rollers,

α = Nominal angle of contact. It is the angle between the line of action of the roller resultant load and a plane perpendicular to the axis of the bearing, and

f_0 = 21.6, for bearings made of hardened steel.

3. For thrust ball bearings, the basic static axial load rating is given by

$$C_0 = f_0.Z.D^2 \sin \alpha$$

where Z = Number of balls carrying thrust in one direction, and

f_0 = 49, for bearings made of hardened steel.

4. For thrust roller bearings, the basic static axial load rating is given by

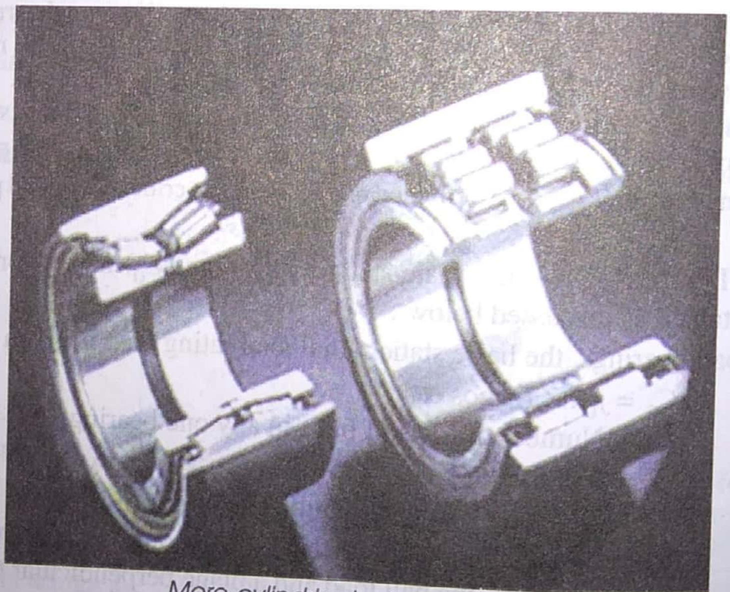
$$C_0 = f_0.Z.l_e.D.\sin \alpha$$

where Z = Number of rollers carrying thrust in one direction, and

f_0 = 98.1, for bearings made of hardened steel.

27.9 Static Equivalent Load for Rolling Contact Bearings

The static equivalent load may be defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied, would cause the same total permanent deformation at the most heavily stressed ball (or roller) and race contact as that which occurs under the actual conditions of loading.



More cylindrical roller bearings

The static equivalent radial load (W_{0R}) for radial or roller bearings under combined radial and axial or thrust loads is given by the greater magnitude of those obtained by the following two equations, *i.e.*

1. $W_{0R} = X_0.W_R + Y_0.W_A$; and 2. $W_{0R} = W_R$

where W_R = Radial load,

W_A = Axial or thrust load,

X_0 = Radial load factor, and

Y_0 = Axial or thrust load factor.

According to IS : 3824 – 1984, the values of X_0 and Y_0 for different bearings are given in the following table :

Table 27.2. Values of X_0 and Y_0 for radial bearings.

S.No.	Type of bearing	Single row bearing		Double row bearing	
		X_0	Y_0	X_0	Y_0
1.	Radial contact groove ball bearings	0.60	0.50	0.60	0.50
2.	Self aligning ball or roller bearings and tapered roller bearing	0.50	0.22 cot θ	1	0.44 cot θ
3.	Angular contact groove bearings :				
	α = 15°	0.50	0.46	1	0.92
	α = 20°	0.50	0.42	1	0.84
	α = 25°	0.50	0.38	1	0.76
	α = 30°	0.50	0.33	1	0.66
	α = 35°	0.50	0.29	1	0.58
	α = 40°	0.50	0.26	1	0.52
	α = 45°	0.50	0.22	1	0.44

Notes : 1. The static equivalent radial load (W_{0R}) is always greater than or equal to the radial load (W_R).

2. For two similar single row angular contact ball bearings, mounted 'face-to-face' or 'back-to-back', use the values of X_0 and Y_0 which apply to a double row angular contact ball bearings. For two or more similar single row angular contact ball bearings mounted 'in tandem', use the values of X_0 and Y_0 which apply to a single row angular contact ball bearings.

3. The static equivalent radial load (W_{0R}) for all cylindrical roller bearings is equal to the radial load (W_R).

4. The static equivalent axial or thrust load (W_{0A}) for thrust ball or roller bearings with angle of contact $\alpha \neq 90°$, under combined radial and axial loads is given by

$$W_{0A} = 2.3\, W_R.\tan\alpha + W_A$$

This formula is valid for all ratios of radial to axial load in the case of direction bearings. For single direction bearings, it is valid where $W_R / W_A \leq 0.44 \cot\alpha$.

5. The thrust ball or roller bearings with $\alpha = 90°$ can support axial loads only. The static equivalent axial load for this type of bearing is given by

$$W_{0A} = W_A$$

27.10 Life of a Bearing

The *life* of an individual ball (or roller) bearing may be defined as the number of revolutions (or hours at some given constant speed) which the bearing runs before the first evidence of fatigue develops in the material of one of the rings or any of the rolling elements.

The ***rating life*** of a group of apparently identical ball or roller bearings is defined as the number of revolutions (or hours at some given constant speed) that 90 per cent of a group of bearings will complete or exceed before the first evidence of fatigue develops (*i.e.* only 10 per cent of a group of bearings fail due to fatigue).

The term ***minimum life*** is also used to denote the rating life. It has been found that the life which 50 per cent of a group of bearings will complete or exceed is approximately 5 times the life which 90 per cent of the bearings will complete or exceed. In other words, we may say that the average life of a bearing is 5 times the rating life (or minimum life). It may be noted that the longest life of a single bearing is seldom longer than the 4 times the average life and the maximum life of a single bearing is about 30 to 50 times the minimum life.

The life of bearings for various types of machines is given in the following table.

Table 27.3. Life of bearings for various types of machines.

S. No.	Application of bearing	Life of bearing, in hours
1.	Instruments and apparatus that are rarely used	500
	(a) Demonstration apparatus, mechanism for operating sliding doors	1000 – 2000
	(b) Aircraft engines	4000 – 8000
2.	Machines used for short periods or intermittently and whose breakdown would not have serious consequences e.g. hand tools, lifting tackle in workshops, and operated machines, agricultural machines, cranes in erecting shops, domestic machines.	8000 – 12 000
3.	Machines working intermittently whose breakdown would have serious consequences e.g. auxillary machinery in power stations, conveyor plant for flow production, lifts, cranes for piece goods, machine tools used frequently.	12 000 – 20 000
4.	Machines working 8 hours per day and not always fully utilised e.g. stationary electric motors, general purpose gear units.	20 000 – 30 000
5.	Machines working 8 hours per day and fully utilised e.g. machines for the engineering industry, cranes for bulk goods, ventilating fans, counter shafts.	40 000 – 60 000
6.	Machines working 24 hours per day e.g. separators, compressors, pumps, mine hoists, naval vessels.	100 000 – 200 000
7.	Machines required to work with high degree of reliability 24 hours per day e.g. pulp and paper making machinery, public power plants, mine-pumps, water works.	

27.11 Basic Dynamic Load Rating of Rolling Contact Bearings

The basic dynamic load rating is defined as the constant stationary radial load (in case of radial ball or roller bearings) or constant axial load (in case of thrust ball or roller bearings) which a group of apparently identical bearings with stationary outer ring can endure for a rating life of one million revolutions (which is equivalent to 500 hours of operation at 33.3 r.p.m.) with only 10 per cent failure.

The basic dynamic load rating (C) in newtons for ball and roller bearings may be obtained as discussed below :

1. According to IS: 3824 (Part 1)– 1983, the basic dynamic radial load rating for radial and angular contact ball bearings, except the filling slot type, with balls not larger than 25.4 mm in diameter, is given by

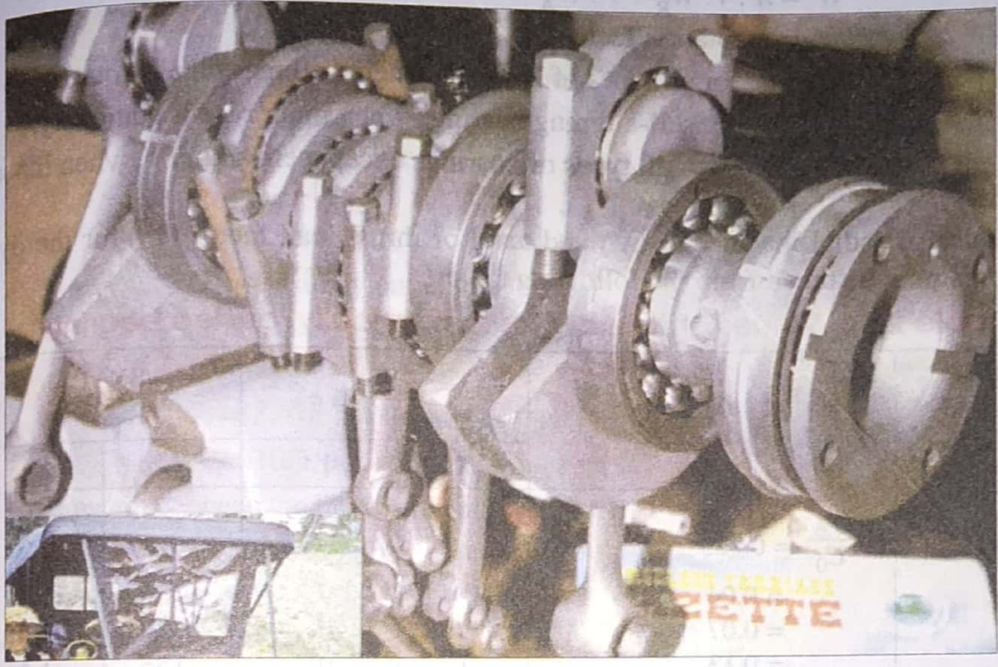
$$C = f_c\ (i \cos \alpha)^{0.7}\ Z^{2/3} \cdot D^{1.8}$$

and for balls larger than 25.4 mm in diameter,

$$C = 3.647\ f_c\ (i \cos \alpha)^{0.7}\ Z^{2/3} \cdot D^{1.4}$$

where f_c = A factor, depending upon the geometry of the bearing components, the accuracy of manufacture and the material used.

and i, Z, D and α have usual meanings as discussed in Art. 27.8.



Ball bearings

2. According to IS: 3824 (Part 2)–1983, the basic dynamic radial load rating for radial roller bearings is given by

$$C = f_c\,(i.l_e \cos\alpha)^{7/9}\, Z^{3/4} .\, D^{29/27}$$

3. According to IS: 3824 (Part 3)–1983, the basic dynamic axial load rating for single row, single or double direction thrust ball bearings is given as follows :

(a) For balls not larger than 25.4 mm in diameter and $\alpha = 90°$,

$$C = f_c \,.\, Z^{2/3} \,.\, D^{1.8}$$

(b) For balls not larger than 25.4 mm in diameter and $\alpha \neq 90°$,

$$C = f_c\,(\cos\alpha)^{0.7} \tan\alpha .\, Z^{2/3} \,.\, D^{1.8}$$

(c) For balls larger than 25.4 mm in diameter and $\alpha = 90°$

$$C = 3.647\, f_c \,.\, Z^{2/3} \,.\, D^{1.4}$$

(d) For balls larger than 25.4 mm in diameter and $\alpha \neq 90°$,

$$C = 3.647\, f_c\,(\cos\alpha)^{0.7} \tan\alpha \,.\, Z^{2/3} \,.\, D^{1.4}$$

4. According to IS: 3824 (Part 4)–1983, the basic dynamic axial load rating for single row, single or double direction thrust roller bearings is given by

$$C = f_c \,.\, l_e^{\,7/9} \,.\, Z^{3/4} \,.\, D^{29/27} \qquad \text{... (when } \alpha = 90°)$$

$$= f_c\,(l_e \cos\alpha)^{7/9} \tan\alpha .Z^{3/4} .\, D^{29/27} \qquad \text{... (when } \alpha \neq 90°)$$

27.12 Dynamic Equivalent Load for Rolling Contact Bearings

The dynamic equivalent load may be defined as the constant stationary radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied to a bearing with rotating inner ring and stationary outer ring, would give the same life as that which the bearing will attain under the actual conditions of load and rotation.

The dynamic equivalent radial load (W) for radial and angular contact bearings, except the filling slot types, under combined constant radial load (W_R) and constant axial or thrust load (W_A) is given by

$$W = X \cdot V \cdot W_R + Y \cdot W_A$$

where V = A rotation factor,

= 1, for all types of bearings when the inner race is rotating,

= 1, for self-aligning bearings when inner race is stationary,

= 1.2, for all types of bearings except self-aligning, when inner race is stationary.

The values of radial load factor (X) and axial or thrust load factor (Y) for the dynamically loaded bearings may be taken from the following table:

Table 27.4. Values of X and Y for dynamically loaded bearings.

Type of bearing	*Specifications*	$\frac{W_A}{W_R} \leq e$		$\frac{W_A}{W_R} > e$		e
		X	Y	X	Y	
Deep groove ball bearing	$\frac{W_A}{C_0} = 0.025$	1	0	0.56	2.0	0.22
	= 0.04				1.8	0.24
	= 0.07				1.6	0.27
	= 0.13				1.4	0.31
	= 0.25				1.2	0.37
	= 0.50				1.0	0.44
Angular contact ball bearings	Single row	1	0	0.35	0.57	1.14
	Two rows in tandem		0	0.35	0.57	1.14
	Two rows back to back		0.55	0.57	0.93	1.14
	Double row		0.73	0.62	1.17	0.86
Self-aligning bearings	Light series : for bores					
	10 – 20 mm	1	1.3	6.5	2.0	0.50
	25 – 35		1.7		2.6	0.37
	40 – 45		2.0		3.1	0.31
	50 – 65		2.3		3.5	0.28
	70 – 100		2.4		3.8	0.26
	105 – 110		2.3		3.5	0.28
	Medium series : for bores					
	12 mm		1.0	0.65	1.6	0.63
	15 – 20		1.2		1.9	0.52
	25 – 50		1.5		2.3	0.43
	55 – 90		1.6		2.5	0.39
Spherical roller bearings	For bores :					
	25 – 35 mm	1	2.1	0.67	3.1	0.32
	40 – 45		2.5		3.7	0.27
	50 – 100		2.9		4.4	0.23
	100 – 200		2.6		3.9	0.26
Taper roller bearings	For bores :					
	30 – 40 mm	1	0	0.4	1.60	0.37
	45 – 110				1.45	0.44
	120 – 150				1.35	0.41

27.13 Dynamic Load Rating for Rolling Contact Bearings under Variable Loads

The approximate rating (or service) life of ball or roller bearings is based on the fundamental equation,

$$L = \left(\frac{C}{W}\right)^k \times 10^6 \text{ revolutions}$$

or

$$C = W\left(\frac{L}{10^6}\right)^{1/k}$$

where
L = Rating life,
C = Basic dynamic load rating,
W = Equivalent dynamic load, and
k = 3, for ball bearings,
= 10/3, for roller bearings.

Roller bearing

The relationship between the life in revolutions (L) and the life in working hours (L_H) is given by

$$L = 60\, N \,.\, L_H \text{ revolutions}$$

where N is the speed in r.p.m.

Now consider a rolling contact bearing subjected to variable loads. Let W_1, W_2, W_3 etc., be the loads on the bearing for successive n_1, n_2, n_3 etc., number of revolutions respectively.

If the bearing is operated exclusively at the constant load W_1, then its life is given by

$$L_1 = \left(\frac{C}{W_1}\right)^k \times 10^6 \text{ revolutions}$$

∴ Fraction of life consumed with load W_1 acting for n_1 number of revolutions is

$$\frac{n_1}{L_1} = n_1\left(\frac{W_1}{C}\right)^k \times \frac{1}{10^6}$$

Similarly, fraction of life consumed with load W_2 acting for n_2 number of revolutions is

$$\frac{n_2}{L_2} = n_2\left(\frac{W_2}{C}\right)^k \times \frac{1}{10^6}$$

and fraction of life consumed with load W_3 acting for n_3 number of revolutions is

$$\frac{n_3}{L_3} = n_3\left(\frac{W_3}{C}\right)^k \times \frac{1}{10^6}$$

But $\frac{n_1}{L_1} + \frac{n_2}{L_2} + \frac{n_3}{L_3} + ... = 1$

or $n_1\left(\frac{W_1}{C}\right)^k \times \frac{1}{10^6} + n_2\left(\frac{W_2}{C}\right)^k \times \frac{1}{10^6} + n_3\left(\frac{W_3}{C}\right)^k \times \frac{1}{10^6} + ... = 1$

$$\therefore n_1(W_1)^k + n_2(W_2)^k + n_3(W_3)^k + ... = C^k \times 10^6 \qquad ...(i)$$

If an equivalent constant load (W) is acting for n number of revolutions, then

$$n = \left(\frac{C}{W}\right)^k \times 10^6$$

$$\therefore \quad n\,(W)^k = C^k \times 10^6$$

where $\quad n = n_1 + n_2 + n_3 + \ldots..$ ...(ii)

From equations (i) and (ii), we have

$$n_1\,(W_1)^k + n_2\,(W_2)^k + n_3\,(W_3)^k + \ldots.. = n\,(W)^k$$

$$\therefore \quad W = \left[\frac{n_1\,(W_1)^k + n_2\,(W_2)^k + n_3\,(W_3)^k + \ldots.}{n}\right]^{1/k}$$

Substituting $n = n_1 + n_2 + n_3 + \ldots...$, and $k = 3$ for ball bearings, we have

$$W = \left[\frac{n_1\,(W_1)^3 + n_2\,(W_2)^3 + n_3\,(W_3)^3 + \ldots.}{n_1 + n_2 + n_3 + \ldots.}\right]^{1/3}$$

Note : The above expression may also be written as

$$W = \left[\frac{L_1\,(W_1)^3 + L_2\,(W_2)^3 + L_3\,(W_3)^3 + \ldots}{L_1 + L_2 + L_3 + \ldots..}\right]^{1/3}$$

See Example 27.6.

27.14 Reliability of a Bearing

We have already discussed in the previous article that the rating life is the life that 90 per cent of a group of identical bearings will complete or exceed before the first evidence of fatigue develops. The reliability (R) is defined as the ratio of the number of bearings which have successfully completed L million revolutions to the total number of bearings under test. Sometimes, it becomes necessary to select a bearing having a reliability of more than 90%. According to Wiebull, the relation between the bearing life and the reliability is given as

$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b \quad \text{or} \quad \frac{L}{a} = \left[\log_e\left(\frac{1}{R}\right)\right]^{1/b} \qquad \ldots(i)$$

where L is the life of the bearing corresponding to the desired reliability R and a and b are constants whose values are

$$a = 6.84, \quad \text{and} \quad b = 1.17$$

If L_{90} is the life of a bearing corresponding to a reliability of 90% (*i.e.* R_{90}), then

$$\frac{L_{90}}{a} = \left[\log_e\left(\frac{1}{R_{90}}\right)\right]^{1/b} \qquad \ldots(ii)$$

Dividing equation (i) by equation (ii), we have

$$\frac{L}{L_{90}} = \left[\frac{\log_e\,(1/R)}{\log_e\,(1/R_{90})}\right]^{1/b} = {}^{*}6.85\,[\log_e\,(1/R)]^{1/1.17} \qquad \ldots(\because b = 1.17)$$

This expression is used for selecting the bearing when the reliability is other than 90%.

Note : If there are n number of bearings in the system each having the same reliability R, then the reliability of the complete system will be

$$R_S = R_p$$

where R_S indicates the probability of one out of p number of bearings failing during its life time.

* $[\log_e\,(1/R_{90})]^{1/b} = [\log_e\,(1/0.90)]^{1/1.17} = (0.10536)^{0.8547} = 0.146$

$$\therefore \quad \frac{L}{L_{90}} = \frac{[\log_e(1/R)]^{1/b}}{0.146} = 6.85\,[\log_e\,(1/R)]^{1/1.17}$$

Bevel Gear

Theory

Bevel Gears

Bevel gears are used to transmit power between two intersecting shafts. There are two types of bevel gears - Straight and spiral. In case of straigth bevel gears, the teeth are straight, which converge into a common apex. In case of spiral bevel gears, the teeth are curved. Straight bevel gears are easy to design and manufacture. These gears produce noise at high speed conditions.

Spiral bevel gears are difficult to design and manufacture. These gears facilitate quieter operations, even at high speeds.

Equivalent or Formative or Virtual number of teeth for bevel gears(Z_v)

Bevel gears are replaced by equivalent spur gears, to simplify the design calculation and analysis. An imaginary spur gear considered in a plane perpendicular to the tooth at the largest end, is called virtual spur gear. The pitch cone radius '*R*' of the bevel gear is equal to the pitch circle radius of the virtual spur gear.

The number of teeth on this imaginary spur gear is called virtual number of teeth.

For pinion For gear

$$Z_v = \frac{Z}{\cos\delta} \qquad Z_{v1} = \frac{Z_1}{\cos\delta_1} \qquad Z_{v2} = \frac{Z_2}{\cos\delta_2}$$

Z_1 & Z_2 are the actual number of teeth on the pinion and gear of the bevel gear respectively.

Z_{v1} & Z_{v2} are the virtual number of teeth on the pinion and gear of the bevel gear respectively.

δ_1 & δ_2 are pitch angles of pinion and gear respectively.

Tooth stress or strength of gears

Lewis Equation

The strength of a bevel gears tooth is obtained in the similar way as that of the spur gear. The modified form of the Lewis equation for the tangential tooth load is given here.

$$F_s = [\sigma_b]\, b \times \pi \times m \times y_v \left(\frac{R-b}{R} \right)$$

$[\sigma_b]$ – Allowable Static Stress

b – Face width

m – Module

y_v – Lewis Factor (or) tooth form factor for the equivalent number of teeth.

R – Cone distance $R = \sqrt{\frac{d_1^2}{2} + \frac{d_2^2}{2}}$

d_1 – Pitch Circle Diameter of pinion

d_2 – Pitch Circle Diameter of gear

$\left(\frac{R-b}{R} \right)$ is a bevel factor.

Dynamic Load for Bevel Gears

Buckingham's Dynamic load

From **PSG** Data book **Pg No. 8.50**

$$F_d = F_t + F_I$$

$$= F_t + \left[\frac{0.164 V_m \times cb + F_t)}{0.164 V_m + 1.485 \sqrt{cb + F_t}} \right]$$

All the notations are similar to spur gear deign.

Wear strength of Bevel Gears

The maximum (or) limiting load of wear strength of bevel gear is given here.

$$F_w = \frac{d_1\, b\, Q_v\, k_w}{\cos\delta}$$

Q_v – Ratio factor based on the virtual number of teeth.

δ – Pitch Angle

b – Face width

k_w – Load stress factor

$$Q_v = \frac{2\, Z_{v2}}{Z_{v1} + Zv_2}$$

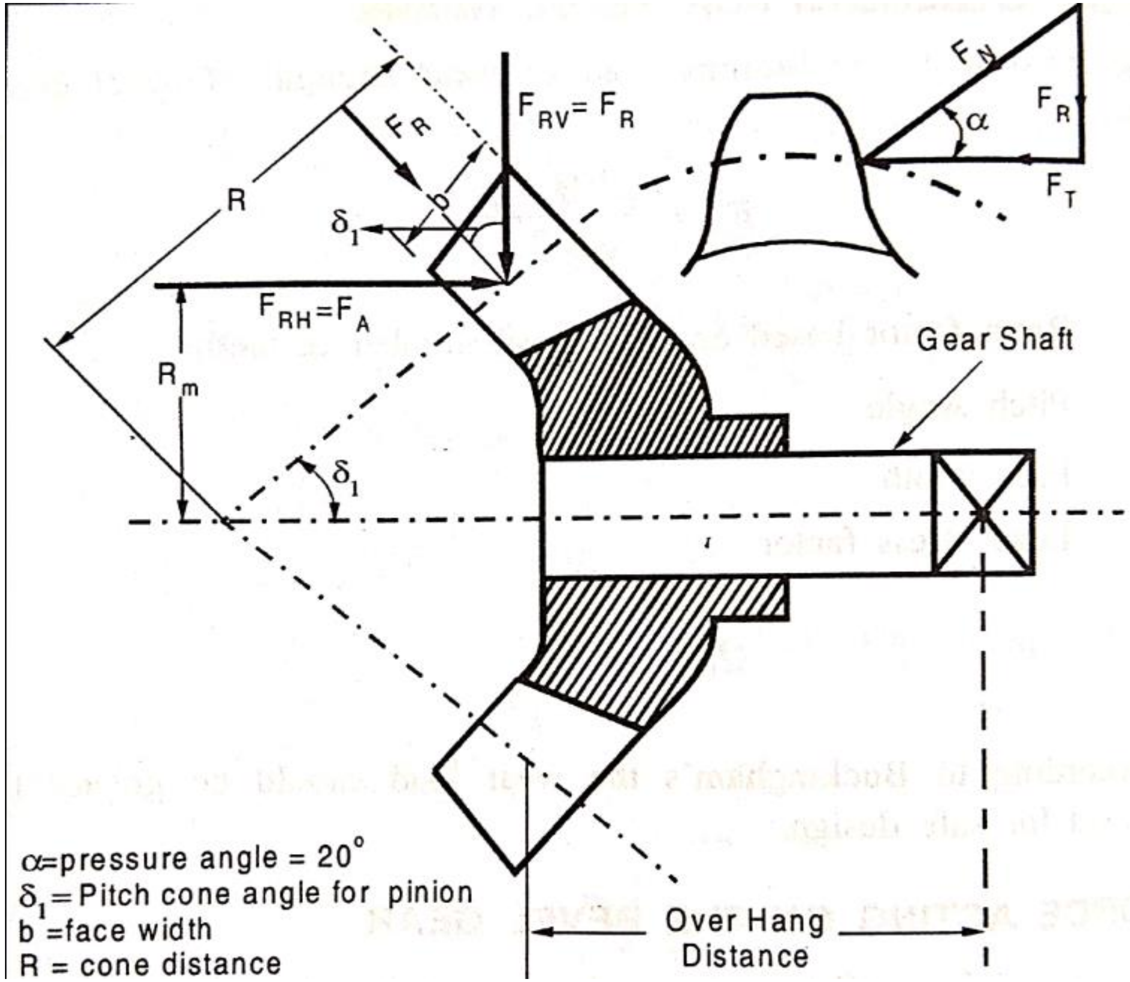
Note: According to Buckingham's the wear load should be greater than dynamic load for safe design.

Forces Acting on Bevel Gear

The normal force (F_N) on the tooth is perpendicular to the tooth profile and thus makes an angle equal to the pressure angle (α) to the pitch circle. Thus the normal force can be resolved into two components.

(i) Tangential component (F_T)

(ii) Radial component (F_R)



The bearing reactions are produced by tangential component (or) tangential tooth load.

The end thrust in the shaft is produced by the radial component.

$$\cos \alpha = \frac{F_T}{F_N}$$

$$F_T = F_N \cos \alpha$$

$$\sin \alpha = \frac{F_R}{F_N}$$

$$F_R = F_N \sin \alpha$$

C H A P T E R

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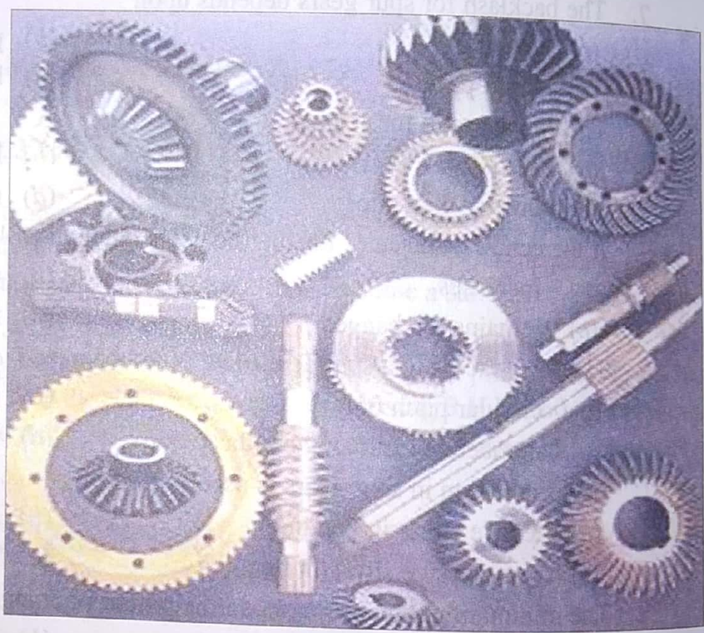
Helical Gears

1. Introduction.
2. Terms used in Helical Gears.
3. Face Width of Helical Gears.
4. Formative or Equivalent Number of Teeth for Helical Gears.
5. Proportions for Helical Gears.
6. Strength of Helical Gears.



29.1 Introduction

A helical gear has teeth in form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears. The helixes may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission.

We have already discussed in Art. 28.4 that the helical gears may be of ***single helical type*** or ***double helical type***. In case of single helical gears there is some axial thrust between the teeth, which is a disadvantage. In order to eliminate this axial thrust, double helical gears (*i.e.*

herringbone gears) are used. It is equivalent to two single helical gears, in which equal and opposite thrusts are provided on each gear and the resulting axial thrust is zero.

29.2 Terms used in Helical Gears

The following terms in connection with helical gears, as shown in Fig. 29.1, are important from the subject point of view.

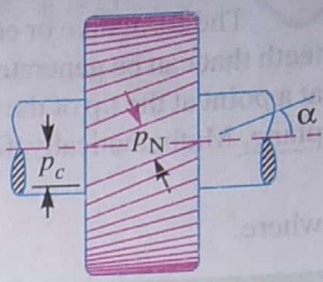


Fig. 29.1. Helical gear (nomenclature).

1. *Helix angle.* It is a constant angle made by the helices with the axis of rotation.

2. *Axial pitch.* It is the distance, parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by p_c. The axial pitch may also be defined as the circular pitch in the plane of rotation or the diametral plane.

3. *Normal pitch.* It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by p_N. The normal pitch may also be defined as the circular pitch in the normal plane which is a plane perpendicular to the teeth. Mathematically, normal pitch,

$$p_N = p_c \cos \alpha$$

Note : If the gears are cut by standard hobs, then the pitch (or module) and the pressure angle of the hob will apply in the normal plane. On the other hand, if the gears are cut by the Fellows gear-shaper method, the pitch and pressure angle of the cutter will apply to the plane of rotation. The relation between the normal pressure angle (ϕ_N) in the normal plane and the pressure angle (ϕ) in the diametral plane (or plane of rotation) is given by

$$\tan \phi_N = \tan \phi \times \cos \alpha$$

29.3 Face Width of Helical Gears

In order to have more than one pair of teeth in contact, the tooth displacement (*i.e.* the advancement of one end of tooth over the other end) or overlap should be atleast equal to the axial pitch, such that

$$\text{Overlap} = p_c = b \tan \alpha \qquad ...(i)$$

The normal tooth load (W_N) has two components ; one is tangential component (W_T) and the other axial component (W_A), as shown in Fig. 29.2. The axial or end thrust is given by

$$W_A = W_N \sin \alpha = W_T \tan \alpha \qquad ...(ii)$$

From equation (*i*), we see that as the helix angle increases, then the tooth overlap increases. But at the same time, the end thrust as given by equation (*ii*), also increases, which is undesirable. It is usually recommended that the overlap should be 15 percent of the circular pitch.

$$\therefore \quad \text{Overlap} = b \tan \alpha = 1.15\, p_c$$

or
$$b = \frac{1.15\, p_c}{\tan \alpha} = \frac{1.15 \times \pi m}{\tan \alpha} \qquad ...(\because p_c = \pi m)$$

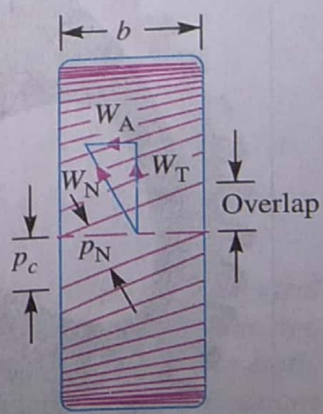


Fig. 29.2. Face width of helical gear.

where
b = Minimum face width, and
m = Module.

Notes : 1. The maximum face width may be taken as 12.5 m to 20 m, where m is the module. In terms of pinion diameter (D_P), the face width should be 1.5 D_P to 2 D_P, although 2.5 D_P may be used.

2. In case of double helical or herringbone gears, the minimum face width is given by

$$b = \frac{2.3\, p_c}{\tan \alpha} = \frac{2.3 \times \pi m}{\tan \alpha}$$

The maximum face width ranges from 20 m to 30 m.

3. In single helical gears, the helix angle ranges from 20° to 35°, while for double helical gears, it may be made upto 45°.

29.4 Formative or Equivalent Number of Teeth for Helical Gears

The formative or equivalent number of teeth for a helical gear may be defined as the number of teeth that can be generated on the surface of a cylinder having a radius equal to the radius of curvature at a point at the tip of the minor axis of an ellipse obtained by taking a section of the gear in the normal plane. Mathematically, formative or equivalent number of teeth on a helical gear,

$$T_E = T / \cos^3 \alpha$$

where T = Actual number of teeth on a helical gear, and

α = Helix angle.

29.5 Proportions for Helical Gears

Though the proportions for helical gears are not standardised, yet the following are recommended by American Gear Manufacturer's Association (AGMA).

Pressure angle in the plane of rotation,

ϕ = 15° to 25°

Helix angle, α = 20° to 45°

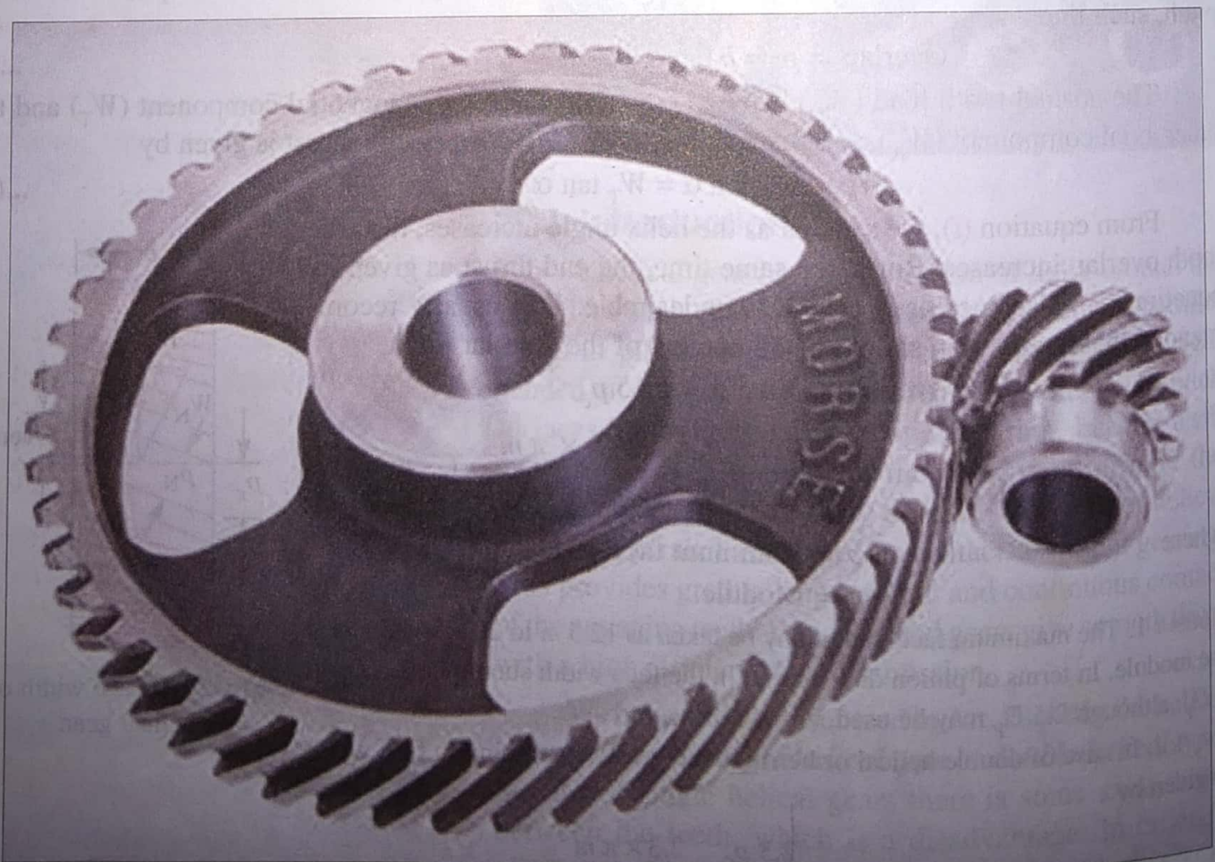
Addendum = 0.8 m (Maximum)

Dedendum = 1 m (Minimum)

Minimum total depth = 1.8 m

Minimum clearance = 0.2 m

Thickness of tooth = 1.5708 m



In helical gears, the teeth are inclined to the axis of the gear.

29.6 Strength of Helical Gears

In helical gears, the contact between mating teeth is gradual, starting at one end and moving along the teeth so that at any instant the line of contact runs diagonally across the teeth. Therefore in order to find the strength of helical gears, a modified Lewis equation is used. It is given by

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y'$$

where
W_T = Tangential tooth load,
σ_o = Allowable static stress,
C_v = Velocity factor,
b = Face width,
m = Module, and
y' = Tooth form factor or Lewis factor corresponding to the formative or virtual or equivalent number of teeth.

Notes : 1. The value of velocity factor (C_v) may be taken as follows :

$$C_v = \frac{6}{6+v}, \text{ for peripheral velocities from 5 m / s to 10 m / s.}$$

$$= \frac{15}{15+v}, \text{ for peripheral velocities from 10 m / s to 20 m / s.}$$

$$= \frac{0.75}{0.75+\sqrt{v}}, \text{ for peripheral velocities greater than 20 m / s.}$$

$$= \frac{0.75}{1+v} + 0.25, \text{ for non-metallic gears.}$$

2. The dynamic tooth load on the helical gears is given by

$$W_D = W_T + \frac{21\, v\, (b.C \cos^2 \alpha + W_T) \cos \alpha}{21\, v + \sqrt{b.C \cos^2 \alpha + W_T}}$$

where v, b and C have usual meanings as discussed in spur gears.

3. The static tooth load or endurance strength of the tooth is given by

$$W_S = \sigma_e.b.\pi\, m.y'$$

4. The maximum or limiting wear tooth load for helical gears is given by

$$W_w = \frac{D_P\, b.Q.K}{\cos^2 \alpha}$$

where D_P, b, Q and K have usual meanings as discussed in spur gears.

In this case,
$$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

where ϕ_N = Normal pressure angle.

Example 29.1. *A pair of helical gears are to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10 000 r.p.m. and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 MPa; determine a suitable module and face width from static strength considerations and check the gears for wear, given σ_{es} = 618 MPa.*

Solution. Given : P = 15 kW = 15 × 10^3 W; ϕ = 20° ; α = 45° ; N_P = 10 000 r.p.m. ; D_P = 80 mm = 0.08 m ; D_G = 320 mm = 0.32 m ; $\sigma_{OP} = \sigma_{OG}$ = 100 MPa = 100 N/mm²; σ_{es} = 618 MPa = 618 N/mm²

Module and face width

Let
m = Module in mm, and
b = Face width in mm.

Since both the pinion and gear are made of the same material (*i.e.* cast steel), therefore the pinion is weaker. Thus the design will be based upon the pinion.

We know that the torque transmitted by the pinion,

$$T = \frac{P \times 60}{2\,\pi\,N_P} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 10\,000} = 14.32 \text{ N-m}$$

∴ *Tangential tooth load on the pinion,

$$W_T = \frac{T}{D_P/2} = \frac{14.32}{0.08/2} = 358 \text{ N}$$

We know that number of teeth on the pinion,

$$T_P = D_P/m = 80/m$$

and formative or equivalent number of teeth for the pinion,

$$T_E = \frac{T_P}{\cos^3\alpha} = \frac{80/m}{\cos^3 45°} = \frac{80/m}{(0.707)^3} = \frac{226.4}{m}$$

∴ Tooth form factor for the pinion for 20° stub teeth,

$$y'_P = 0.175 - \frac{0.841}{T_E} = 0.175 - \frac{0.841}{226.4/m} = 0.175 - 0.0037\,m$$

We know that peripheral velocity,

$$v = \frac{\pi\,D_P\,.N_P}{60} = \frac{\pi \times 0.08 \times 10000}{60} = 42 \text{ m/s}$$

∴ Velocity factor,

$$C_v = \frac{0.75}{0.75 + \sqrt{v}} = \frac{0.75}{0.75 + \sqrt{42}} = 0.104 \qquad \text{...(∵ } v \text{ is greater than 20 m/s)}$$

Since the maximum face width (b) for helical gears may be taken as 12.5 m to 20 m, where m is the module, therefore let us take

$$b = 12.5\,m$$

We know that the tangential tooth load (W_T),

$$358 = (\sigma_{OP}\,.\,C_v)\,b.\pi\,m.y'_P$$
$$= (100 \times 0.104)\;12.5\,m \times \pi\,m\,(0.175 - 0.0037\,m)$$
$$= 409\,m^2\,(0.175 - 0.0037\,m) = 72\,m^2 - 1.5\,m^3$$

Solving this expression by hit and trial method, we find that

$$m = 2.3 \text{ say } 2.5 \text{ mm} \quad \textbf{Ans.}$$

and face width, $b = 12.5\,m = 12.5 \times 2.5 = 31.25$ say 32 mm **Ans.**

Checking the gears for wear

We know that velocity ratio,

$$V.R. = \frac{D_G}{D_P} = \frac{320}{80} = 4$$

∴ Ratio factor,

$$Q = \frac{2 \times V.R.}{V.R. + 1} = \frac{2 \times 4}{4 + 1} = 1.6$$

We know that $\tan\phi_N = \tan\phi\cos\alpha = \tan 20° \times \cos 45° = 0.2573$

∴ $\phi_N = 14.4°$

* The tangential tooth load on the pinion may also be obtained by using the relation,

$$W_T = \frac{P}{v}, \text{ where } v = \frac{\pi\,D_P\,.N_P}{60} \text{ (in m/s)}$$

The picture shows double helical gears which are also called herringbone gears.

Since both the gears are made of the same material (*i.e.* cast steel), therefore let us take

$$E_P = E_G = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$$

∴ Load stress factor,

$$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

$$= \frac{(618)^2 \sin 14.4°}{1.4}\left(\frac{1}{200 \times 10^3} + \frac{1}{200 \times 10^3}\right) = 0.678 \text{ N/mm}^2$$

We know that the maximum or limiting load for wear,

$$W_w = \frac{D_P\, b.Q.K}{\cos^2 \alpha} = \frac{80 \times 32 \times 1.6 \times 0.678}{\cos^2 45°} = 5554 \text{ N}$$

Since the maximum load for wear is much more than the tangential load on the tooth, therefore the design is satisfactory from consideration of wear.

Example 29.2. *A helical cast steel gear with 30° helix angle has to transmit 35 kW at 1500 r.p.m. If the gear has 24 teeth, determine the necessary module, pitch diameter and face width for 20° full depth teeth. The static stress for cast steel may be taken as 56 MPa. The width of face may be taken as 3 times the normal pitch. What would be the end thrust on the gear? The tooth factor for 20° full depth involute gear may be taken as* $0.154 - \frac{0.912}{T_E}$, *where* T_E *represents the equivalent number of teeth.*

Solution. Given : $\alpha = 30°$; $P = 35$ kW $= 35 \times 10^3$ W ; $N = 1500$ r.p.m. ; $T_G = 24$; $\phi = 20°$; $\sigma_o = 56$ MPa $= 56$ N/mm² ; $b = 3 \times$ Normal pitch $= 3\, p_N$

Module

Let m = Module in mm, and

D_G = Pitch circle diameter of the gear in mm.

We know that torque transmitted by the gear,

$$T = \frac{P \times 60}{2\,\pi\, N} = \frac{35 \times 10^3 \times 60}{2\,\pi \times 1500} = 223 \text{ N-m} = 223 \times 10^3 \text{ N-mm}$$

C H A P T E R

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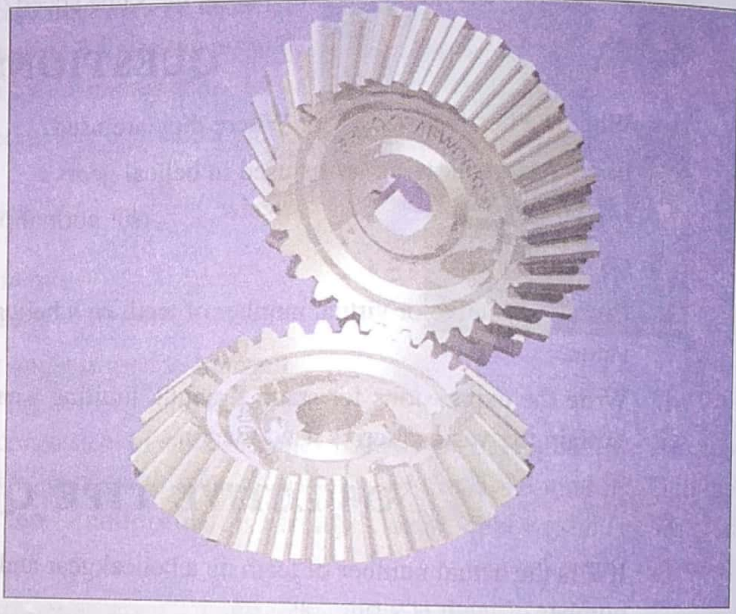
Bevel Gears

1. Introduction.
2. Classification of Bevel Gears.
3. Terms used in Bevel Gears.
4. Determination of Pitch Angle for Bevel Gears.
5. Proportions for Bevel Gears.
6. Formative or Equivalent Number of Teeth for Bevel Gears—Tredgold's Approximation.
7. Strength of Bevel Gears.
8. Forces Acting on a Bevel Gear.
9. Design of a Shaft for Bevel Gears.



30.1 Introduction

The bevel gears are used for transmitting power at a constant velocity ratio between two shafts whose axes intersect at a certain angle. The pitch surfaces for the bevel gear are frustums of cones. The two pairs of cones in contact is shown in Fig. 30.1. The elements of the cones, as shown in Fig. 30.1 (*a*), intersect at the point of intersection of the axis of rotation. Since the radii of both the gears are proportional to their distances from the apex, therefore the cones may roll together without sliding. In Fig. 30.1 (*b*), the elements of both cones do not intersect at the point of shaft intersection. Consequently, there may be pure rolling at only one point of contact and there must be tangential sliding at all other points of contact. Therefore, these cones, cannot be used as pitch surfaces because it is impossible to have positive driving and sliding in the same direction at the same time. We, thus, conclude that the elements of bevel gear pitch cones and shaft axes must intersect at the same point.

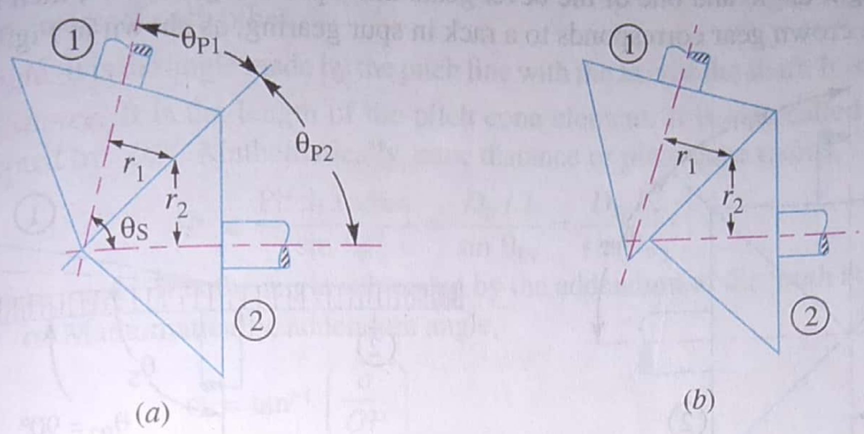


Fig. 30.1. Pitch surface for bevel gears.

The bevel gear is used to change the axis of rotational motion. By using gears of differing numbers of teeth, the speed of rotation can also be changed.

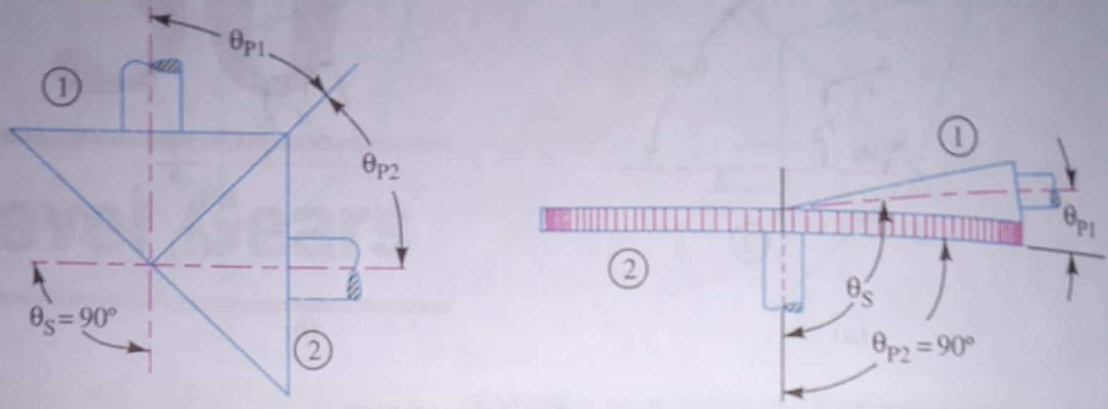
30.2 Classification of Bevel Gears

The bevel gears may be classified into the following types, depending upon the angles between the shafts and the pitch surfaces.

1. *Mitre gears.* When equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle, as shown in Fig. 30.2 (*a*), then they are known as ***mitre gears.***

2. *Angular bevel gears.* When the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as ***angular bevel gears.***

3. *Crown bevel gears.* When the bevel gears connect two shafts whose axes intersect at an angle greater than a right angle and one of the bevel gears has a pitch angle of 90°, then it is known as a crown gear. The crown gear corresponds to a rack in spur gearing, as shown in Fig. 30.2 (*b*).



(*a*) Mitre gears. (*b*) Crown bevel gear.

Fig. 30.2. Classification of bevel gears.

4. *Internal bevel gears.* When the teeth on the bevel gear are cut on the inside of the pitch cone, then they are known as *internal bevel gears.*

Note : The bevel gears may have straight or spiral teeth. It may be assumed, unless otherwise stated, that the bevel gear has straight teeth and the axes of the shafts intersect at right angle.

30.3 Terms used in Bevel Gears

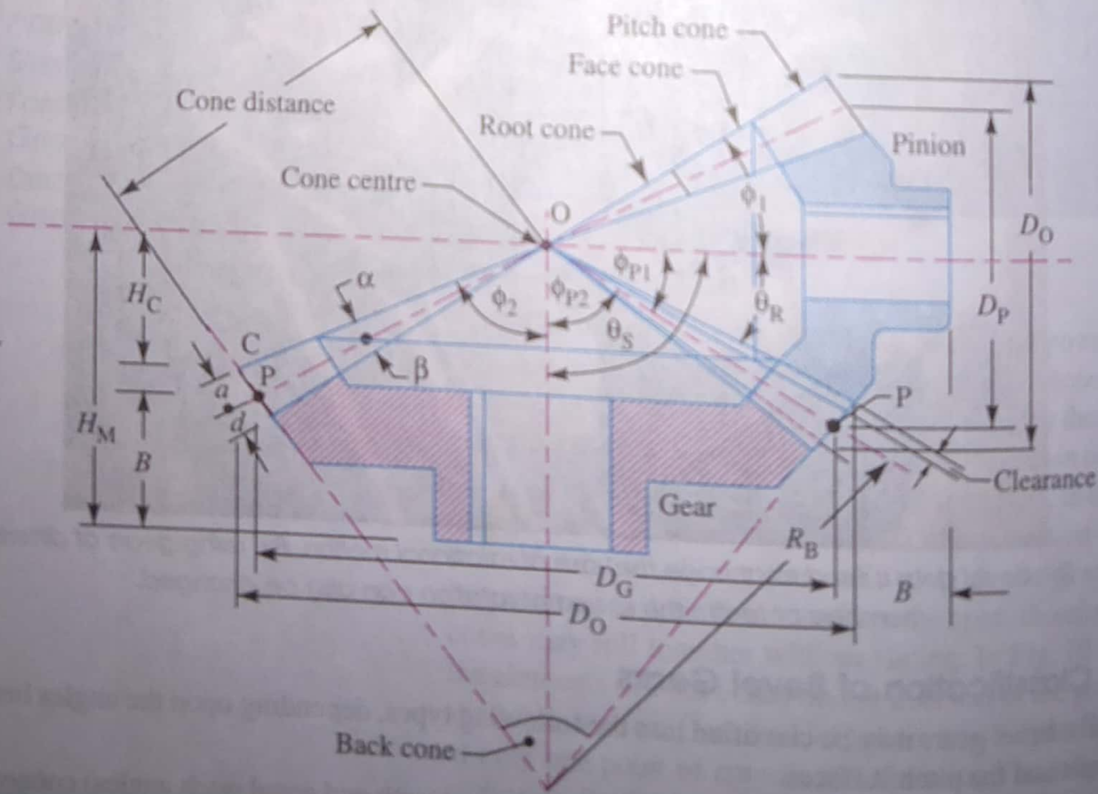


Fig. 30.3. Terms used in bevel gears.

A sectional view of two bevel gears in mesh is shown in Fig. 30.3. The following terms in connection with bevel gears are important from the subject point of view :

1. *Pitch cone.* It is a cone containing the pitch elements of the teeth.

2. *Cone centre.* It is the apex of the pitch cone. It may be defined as that point where the axes of two mating gears intersect each other.

3. *Pitch angle.* It is the angle made by the pitch line with the axis of the shaft. It is denoted by 'θ_P'.

4. *Cone distance.* It is the length of the pitch cone element. It is also called as a ***pitch cone radius***. It is denoted by 'OP'. Mathematically, cone distance or pitch cone radius,

$$OP = \frac{\text{Pitch radius}}{\sin \theta_P} = \frac{D_P/2}{\sin \theta_{P1}} = \frac{D_G/2}{\sin \theta_{P2}}$$

5. *Addendum angle.* It is the angle subtended by the addendum of the tooth at the cone centre. It is denoted by 'α' Mathematically, addendum angle,

$$\alpha = \tan^{-1}\left(\frac{a}{OP}\right)$$

where a = Addendum, and OP = Cone distance.

6. *Dedendum angle.* It is the angle subtended by the dedendum of the tooth at the cone centre. It is denoted by 'β'. Mathematically, dedendum angle,

$$\beta = \tan^{-1}\left(\frac{d}{OP}\right)$$

where d = Dedendum, and OP = Cone distance.

7. *Face angle.* It is the angle subtended by the face of the tooth at the cone centre. It is denoted by 'ϕ'. The face angle is equal to the pitch angle *plus* addendum angle.

8. *Root angle.* It is the angle subtended by the root of the tooth at the cone centre. It is denoted by 'θ_R'. It is equal to the pitch angle *minus* dedendum angle.

9. *Back (or normal) cone.* It is an imaginary cone, perpendicular to the pitch cone at the end of the tooth.

10. **Back cone distance.** It is the length of the back cone. It is denoted by 'R_B'. It is also called back cone radius.

11. *Backing.* It is the distance of the pitch point (P) from the back of the boss, parallel to the pitch point of the gear. It is denoted by 'B'.

12. *Crown height.* It is the distance of the crown point (C) from the cone centre (O), parallel to the axis of the gear. It is denoted by 'H_C'.

13. *Mounting height.* It is the distance of the back of the boss from the cone centre. It is denoted by 'H_M'.

14. *Pitch diameter.* It is the diameter of the largest pitch circle.

15. *Outside or addendum cone diameter.* It is the maximum diameter of the teeth of the gear. It is equal to the diameter of the blank from which the gear can be cut. Mathematically, outside diameter,

$$D_O = D_P + 2\,a \cos \theta_P$$

where D_P = Pitch circle diameter,

a = Addendum, and

θ_P = Pitch angle.

16. *Inside or dedendum cone diameter.* The inside or the dedendum cone diameter is given by

$$D_d = D_P - 2d \cos \theta_P$$

where D_d = Inside diameter, and

d = Dedendum.

D_G = Pitch diameter of the gear, and

D_P = Pitch diameter of the pinion.

Notes : 1. The factor $\left(\frac{L-b}{L}\right)$ may be called as ***bevel factor***.

2. For satisfactory operation of the bevel gears, the face width should be from 6.3 *m* to 9.5 *m*, where *m* is the module. Also the ratio L/b should not exceed 3. For this, the number of teeth in the pinion must not less than $\frac{48}{\sqrt{1+(V.R.)^2}}$, where *V.R.* is the required velocity ratio.

3. The dynamic load for bevel gears may be obtained in the similar manner as discussed for spur gears.

4. The static tooth load or endurance strength of the tooth for bevel gears is given by

$$W_S = \sigma_e.b.\pi\, m.y' \left(\frac{L-b}{L}\right)$$

The value of flexural endurance limit (σ_e) may be taken from Table 28.8, in spur gears.

5. The maximum or limiting load for wear for bevel gears is given by

$$W_w = \frac{D_P.b.Q.K}{\cos\theta_{P1}}$$

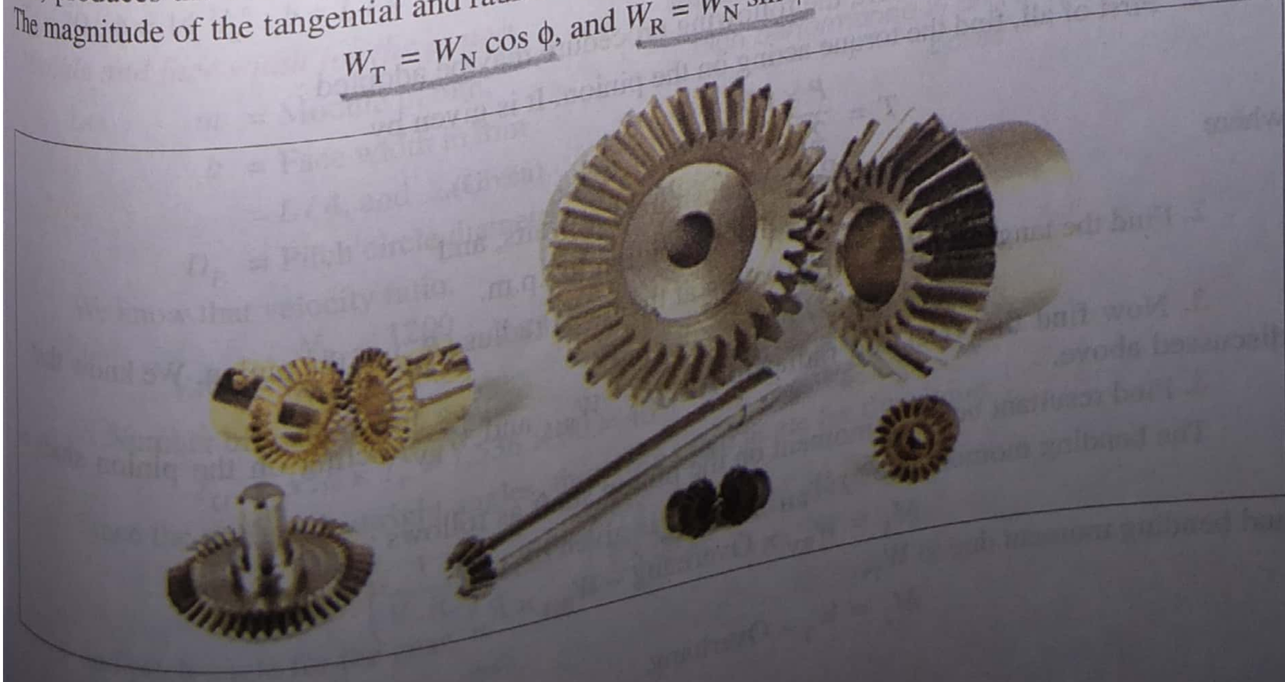
where D_P, *b*, *Q* and *K* have usual meanings as discussed in spur gears except that *Q* is based on formative or equivalent number of teeth, such that

$$Q = \frac{2\,T_{EG}}{T_{EG}+T_{EP}}$$

30.8 Forces Acting on a Bevel Gear

Consider a bevel gear and pinion in mesh as shown in Fig. 30.5. The normal force (W_N) on the tooth is perpendicular to the tooth profile and thus makes an angle equal to the pressure angle (ϕ) to the pitch circle. Thus normal force can be resolved into two components, one is the tangential component (W_T) and the other is the radial component (W_R). The tangential component (*i.e.* the tangential tooth load) produces the bearing reactions while the radial component produces end thrust in the shafts. The magnitude of the tangential and radial components is as follows :

$$W_T = W_N \cos\phi, \text{ and } W_R = W_N \sin\phi = W_T \tan\phi \quad \text{...}(i)$$



These forces are considered to act at the mean radius (R_m). From the geometry of the Fig. 30.5, we find that

$$R_m = \left(L - \frac{b}{2}\right)\sin\theta_{P1} = \left(L - \frac{b}{2}\right)\frac{D_P}{2L} \quad \ldots \left(\because \sin\theta_{P1} = \frac{D_P/2}{L}\right)$$

Now the radial force (W_R) acting at the mean radius may be further resolved into two components, W_{RH} and W_{RV}, in the axial and radial directions as shown in Fig. 30.5. Therefore the axial force acting on the pinion shaft,

$$W_{RH} = W_R \sin\theta_{P1} = W_T \tan\phi \,.\, \sin\theta_{P1} \quad \ldots\text{[From equation } (i)\text{]}$$

and the radial force acting on the pinion shaft,

$$W_{RV} = W_R \cos\theta_{P1} = W_T \tan\phi \,.\, \cos\theta_{P1}$$

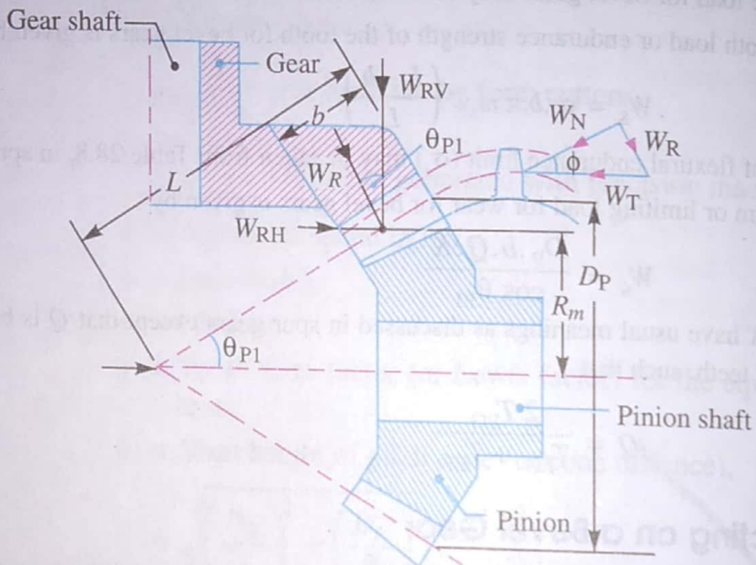


Fig. 30.5. Forces acting on a bevel gear.

A little consideration will show that the axial force on the pinion shaft is equal to the radial force on the gear shaft but their directions are opposite. Similarly, the radial force on the pinion shaft is equal to the axial force on the gear shaft, but act in opposite directions.

30.9 Design of a Shaft for Bevel Gears

In designing a pinion shaft, the following procedure may be adopted :

1. First of all, find the torque acting on the pinion. It is given by

$$T = \frac{P \times 60}{2\pi N_P} \text{ N-m}$$

where
P = Power transmitted in watts, and
N_P = Speed of the pinion in r.p.m.

2. Find the tangential force (W_T) acting at the mean radius (R_m) of the pinion. We know that

$$W_T = T / R_m$$

3. Now find the axial and radial forces (*i.e.* W_{RH} and W_{RV}) acting on the pinion shaft as discussed above.

4. Find resultant bending moment on the pinion shaft as follows :

The bending moment due to W_{RH} and W_{RV} is given by

$$M_1 = W_{RV} \times \text{Overhang} - W_{RH} \times R_m$$

and bending moment due to W_T,

$$M_2 = W_T \times \text{Overhang}$$

∴ Resultant bending moment,

$$M = \sqrt{(M_1)^2 + (M_2)^2}$$

5. Since the shaft is subjected to twisting moment (T) and resultant bending moment (M), therefore equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2}$$

6. Now the diameter of the pinion shaft may be obtained by using the torsion equation. We know that

$$T_e = \frac{\pi}{16} \times \tau\,(d_P)^3$$

where

d_P = Diameter of the pinion shaft, and

τ = Shear stress for the material of the pinion shaft.

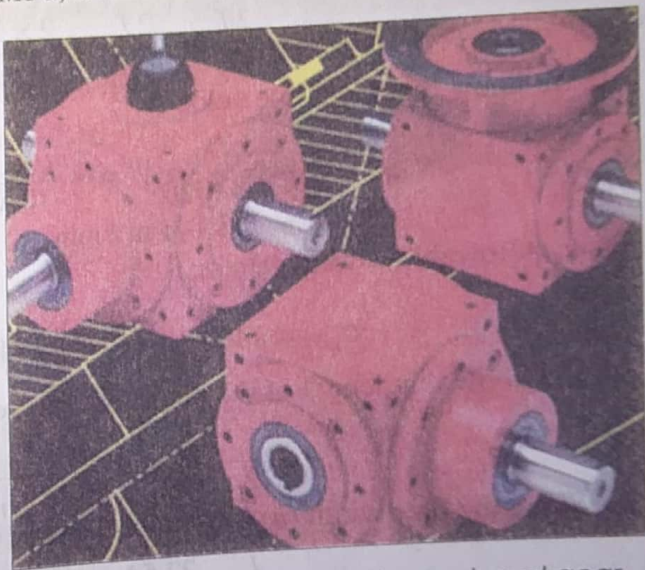
7. The same procedure may be adopted to find the diameter of the gear shaft.

Example 30.1. *A 35 kW motor running at 1200 r.p.m. drives a compressor at 780 r.p.m. through a 90° bevel gearing arrangement. The pinion has 30 teeth. The pressure angle of teeth is $14^1/_2°$. The wheels are capable of withstanding a dynamic stress,*

$$\sigma_w = 140\left(\frac{280}{280+v}\right) \text{ MPa, where } v \text{ is the pitch line speed in m / min.}$$

The form factor for teeth may be taken as $0.124 - \frac{0.686}{T_E}$, where T_E is the number of teeth equivalent of a spur gear.

The face width may be taken as $\frac{1}{4}$ of the slant height of pitch cone. Determine for the pinion, the module pitch, face width, addendum, dedendum, outside diameter and slant height.



High performance 2- and 3 -way bevel gear boxes

Solution : Given : $P = 35$ kW $= 35 \times 10^3$ W ; $N_P = 1200$ r.p.m. ; $N_G = 780$ r.p.m. ; $\theta_S = 90°$; $T_P = 30$; $\phi = 14\,^1/_2°$; $b = L/4$

Module and face width for the pinion

Let m = Module in mm,

b = Face width in mm

$= L/4$, and ...(Given)

D_P = Pitch circle diameter of the pinion.

We know that velocity ratio,

$$V.R. = \frac{N_P}{N_G} = \frac{1200}{780} = 1.538$$

∴ Number of teeth on the gear,

$$T_G = V.R. \times T_P = 1.538 \times 30 = 46$$

Since the shafts are at right angles, therefore pitch angle for the pinion,

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R.}\right) = \tan^{-1}\left(\frac{1}{1.538}\right) = \tan^{-1}(0.65) = 33°$$

and pitch angle for the gear,

$$\theta_{P2} = 90° - 33° = 57°$$

We know that formative number of teeth for pinion,

$$T_{EP} = T_P.\sec \theta_{P1} = 30 \times \sec 33° = 35.8$$

and formative number of teeth for the gear,

$$T_{EG} = T_G.\sec \theta_{P2} = 46 \times \sec 57° = 84.4$$

Tooth form factor for the pinion

$$y'_P = 0.124 - \frac{0.686}{T_{EP}} = 0.124 - \frac{0.686}{35.8} = 0.105$$

and tooth form factor for the gear,

$$y'_G = 0.124 - \frac{0.686}{T_{EG}} = 0.124 - \frac{0.686}{84.4} = 0.116$$

Since the allowable static stress (σ_o) for both the pinion and gear is same (*i.e.* 140 MPa or N/mm^2) and y'_P is less than y'_G, therefore the pinion is weaker. Thus the design should be based upon the pinion.

We know that the torque on the pinion,

$$T = \frac{P \times 60}{2\pi N_P} = \frac{35 \times 10^3 \times 60}{2\pi \times 1200} = 278.5 \text{ N-m} = 278\,500 \text{ N-mm}$$

∴ Tangential load on the pinion,

$$W_T = \frac{2T}{D_P} = \frac{2T}{m.T_P} = \frac{2 \times 278\,500}{m \times 30} = \frac{18\,567}{m} \text{ N}$$

We know that pitch line velocity,

$$v = \frac{\pi D_P.N_P}{1000} = \frac{\pi m.T_P.N_P}{1000} = \frac{\pi m \times 30 \times 1200}{1000} \text{ m/min}$$
$$= 113.1\, m \text{ m/min}$$

∴ Allowable working stress,

$$\sigma_w = 140\left(\frac{280}{280 + v}\right) = 140\left(\frac{280}{280 + 113.1m}\right) \text{ MPa or N/mm}^2$$

We know that length of the pitch cone element or slant height of the pitch cone,

$$L = \frac{D_P}{2 \sin \theta_{P1}} = \frac{m \times T_P}{2 \sin \theta_{P1}} = \frac{m \times 30}{2 \sin 33°} = 27.54\, m \text{ mm}$$

Since the face width (b) is 1/4th of the slant height of the pitch cone, therefore

$$b = \frac{L}{4} = \frac{27.54m}{4} = 6.885\, m \text{ mm}$$

We know that tangential load on the pinion,

$$W_T = (\sigma_{OP} \times C_v)\, b.\pi\, m.y'_P \left(\frac{L - b}{L}\right)$$
$$= \sigma_w.b.\pi\, m.y'_P \left(\frac{L - b}{L}\right) \qquad \ldots (\because \sigma_w = \sigma_{OP} \times C_v)$$

or
$$\frac{18\,567}{m} = 140\left(\frac{280}{280 + 113.1m}\right) 6.885\, m \times \pi\, m \times 0.105 \left(\frac{27.54m - 6.885m}{27.54m}\right)$$
$$= \frac{66\,780 m^2}{280 + 113.1m}$$

or
$$280 + 113.1\, m = 66\,780\, m^2 \times \frac{m}{18\,567} = 3.6\, m^3$$

Solving this expression by hit and trial method, we find that

$$m = 6.6 \text{ say } 8 \text{ mm } \textbf{Ans.}$$

and face width,

$$b = 6.885\, m = 6.885 \times 8 = 55 \text{ mm } \textbf{Ans.}$$

Addendum and dedendum for the pinion

We know that addendum,

$$a = 1\, m = 1 \times 8 = 8 \text{ mm } \textbf{Ans.}$$

and dedendum,

$$d = 1.2\, m = 1.2 \times 8 = 9.6 \text{ mm } \textbf{Ans.}$$

Outside diameter for the pinion

We know that outside diameter for the pinion,

$$D_O = D_P + 2\,a \cos \theta_{P1} = m.T_P + 2\,a \cos \theta_{P1} \quad \ldots (\because D_P = m\,.\,T_P)$$

$$= 8 \times 30 + 2 \times 8 \cos 33° = 253.4 \text{ mm } \textbf{Ans.}$$

Slant height

We know that slant height of the pitch cone,

$$L = 27.54\, m = 27.54 \times 8 = 220.3 \text{ mm } \textbf{Ans.}$$

Example 30.2. *A pair of cast iron bevel gears connect two shafts at right angles. The pitch diameters of the pinion and gear are 80 mm and 100 mm respectively. The tooth profiles of the gears are of 14 ½° composite form. The allowable static stress for both the gears is 55 MPa. If the pinion transmits 2.75 kW at 1100 r.p.m., find the module and number of teeth on each gear from the standpoint of strength and check the design from the standpoint of wear. Take surface endurance limit as 630 MPa and modulus of elasticity for cast iron as 84 kN/mm².*

Solution. Given : $\theta_S = 90°$; $D_P = 80$ mm $= 0.08$ m ; $D_G = 100$ mm $= 0.1$ m ; $\phi = 14\frac{1}{2}°$; $\sigma_{OP} = \sigma_{OG} = 55$ MPa $= 55$ N/mm² ; $P = 2.75$ kW $= 2750$ W ; $N_P = 1100$ r.p.m. ; $\sigma_{es} = 630$ MPa $= 630$ N/mm² ; $E_P = E_G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

Module

Let m = Module in mm.

Since the shafts are at right angles, therefore pitch angle for the pinion,

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R.}\right) = \tan^{-1}\left(\frac{D_P}{D_G}\right) = \tan^{-1}\left(\frac{80}{100}\right) = 38.66°$$

and pitch angle for the gear,

$$\theta_{P2} = 90° - 38.66° = 51.34°$$

We know that formative number of teeth for pinion,

$$T_{EP} = T_P\,.\sec \theta_{P1} = \frac{80}{m} \times \sec 38.66° = \frac{102.4}{m} \quad \ldots (\because T_P = D_P / m)$$

and formative number of teeth on the gear,

$$T_{EG} = T_G\,.\sec \theta_{P2} = \frac{100}{m} \times \sec 51.34° = \frac{160}{m} \quad \ldots (\because T_G = D_G / m)$$

Since both the gears are made of the same material, therefore pinion is the weaker. Thus the design should be based upon the pinion.

We know that tooth form factor for the pinion having 14 ½° composite teeth,

$$y'_P = 0.124 - \frac{0.684}{T_{EP}} = 0.124 - \frac{0.684 \times m}{102.4}$$

$$= 0.124 - 0.006\,68\, m$$

and pitch line velocity,

$$v = \frac{\pi D_P\,.N_P}{60} = \frac{\pi \times 0.08 \times 1100}{60} = 4.6 \text{ m/s}$$

Taking velocity factor,

$$C_v = \frac{6}{6+v} = \frac{6}{6+4.6} = 0.566$$

We know that length of the pitch cone element or slant height of the pitch cone,

$$*L = \sqrt{\left(\frac{D_G}{2}\right)^2 + \left(\frac{D_P}{2}\right)^2} = \sqrt{\left(\frac{100}{2}\right)^2 + \left(\frac{80}{2}\right)^2} = 64 \text{ mm}$$

Assuming the face width (b) as 1/3rd of the slant height of the pitch cone (L), therefore

$$b = L/3 = 64/3 = 21.3 \text{ say } 22 \text{ mm}$$

We know that torque on the pinion,

$$T = \frac{P \times 60}{2\pi \times N_P} = \frac{2750 \times 60}{2\pi \times 1100} = 23.87 \text{ N-m} = 23\,870 \text{ N-mm}$$

∴ Tangential load on the pinion,

$$W_T = \frac{T}{D_P/2} = \frac{23\,870}{80/2} = 597 \text{ N}$$

We also know that tangential load on the pinion,

$$W_T = (\sigma_{OP} \times C_v)\, b \times \pi\, m \times y'_P \left(\frac{L-b}{L}\right)$$

or
$$597 = (55 \times 0.566)\, 22 \times \pi\, m\, (0.124 - 0.00\,668\, m) \left(\frac{64-22}{64}\right)$$

$$= 1412\, m\, (0.124 - 0.006\,68\, m)$$

$$= 175\, m - 9.43\, m^2$$

Solving this expression by hit and trial method, we find that

$$m = 4.5 \text{ say } 5 \text{ mm } \textbf{Ans.}$$

The bevel gear turbine

Number of teeth on each gear

We know that number of teeth on the pinion,

$$T_P = D_P/m = 80/5 = 16 \textbf{ Ans.}$$

and number of teeth on the gear,

$$T_G = D_G/m = 100/5 = 20 \textbf{ Ans.}$$

Checking the gears for wear

We know that the load-stress factor,

$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

$$= \frac{(630)^2 \sin 14\,^1/_2\,°}{1.4}\left[\frac{1}{84 \times 10^3} + \frac{1}{84 \times 10^3}\right] = 1.687$$

and ratio factor, $Q = \dfrac{2\,T_{EG}}{T_{EG} + T_{EP}} = \dfrac{2 \times 160/m}{160/m + 102.4/m} = 1.22$

* The length of the pitch cone element (L) may also obtained by using the relation

$$L = D_P / 2 \sin\theta_{P1}$$

∴ Maximum or limiting load for wear,

$$W_w = \frac{D_P \cdot b \cdot Q \cdot K}{\cos\theta_{P1}} = \frac{80 \times 22 \times 1.22 \times 1.687}{\cos 38.66^\circ} = 4640 \text{ N}$$

Since the maximum load for wear is much more than the tangential load (W_T), therefore the design is satisfactory from the consideration of wear. **Ans.**

Example 30.3. *A pair of bevel gears connect two shafts at right angles and transmits 9 kW. Determine the required module and gear diameters for the following specifications :*

Particulars	*Pinion*	*Gear*
Number of teeth	*21*	*60*
Material	*Semi-steel*	*Grey cast iron*
Brinell hardness number	*200*	*160*
Allowable static stress	*85 MPa*	*55 MPa*
Speed	*1200 r.p.m.*	*420 r.p.m.*
Tooth profile	$14\frac{1}{2}^\circ$ *composite*	$14\frac{1}{2}^\circ$ *composite*

Check the gears for dynamic and wear loads.

Solution. Given : $\theta_S = 90^\circ$; $P = 9$ kW $= 9000$ W ; $T_P = 21$; $T_G = 60$; $\sigma_{OP} = 85$ MPa $= 85$ N/mm^2 ; $\sigma_{OG} = 55$ MPa $= 55$ N/mm^2 ; $N_P = 1200$ r.p.m. ; $N_G = 420$ r.p.m. ; $\phi = 14\ ^1/_2{}^\circ$

Required module

Let m = Required module in mm.

Since the shafts are at right angles, therefore pitch angle for the pinion,

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R.}\right) = \tan^{-1}\left(\frac{T_P}{T_G}\right) = \tan^{-1}\left(\frac{21}{60}\right) = 19.3^\circ$$

and pitch angle for the gear,

$$\theta_{P2} = \theta_S - \theta_{P1} = 90^\circ - 19.3^\circ = 70.7^\circ$$

We know that formative number of teeth for the pinion,

$$T_{EP} = T_P \cdot \sec\theta_{P1} = 21 \sec 19.3^\circ = 22.26$$

and formative number of teeth for the gear,

$$T_{EG} = T_G \cdot \sec\theta_{P2} = 60 \sec 70.7^\circ = 181.5$$

We know that tooth form factor for the pinion,

$$y'_P = 0.124 - \frac{0.684}{T_{EP}} = 0.124 - \frac{0.684}{22.26} = 0.093$$

... (For $14\ ^1/_2{}^\circ$ composite system)

and tooth form factor for the gear,

$$y'_G = 0.124 - \frac{0.684}{T_{EG}} = 0.124 - \frac{0.684}{181.5} = 0.12$$

∴ $\sigma_{OP} \times y'_P = 85 \times 0.093 = 7.905$

and $\sigma_{OG} \times y'_G = 55 \times 0.12 = 6.6$

Since the product $\sigma_{OG} \times y'_G$ is less than $\sigma_{OP} \times y'_P$, therefore the gear is weaker. Thus, the design should be based upon the gear.

We know that torque on the gear,

$$T = \frac{P \times 60}{2\pi N_G} = \frac{9000 \times 60}{2\pi \times 420} = 204.6 \text{ N-m} = 204\,600 \text{ N-mm}$$

CHAPTER

31

Worm Gears

31.1 Introduction

The worm gears are widely used for transmitting power at high velocity ratios between non-intersecting shafts that are generally, but not necessarily, at right angles. It can give velocity ratios as high as 300 : 1 or more in a single step in a minimum of space, but it has a lower efficiency. The worm gearing is mostly used as a speed reducer, which consists of worm and a worm wheel or gear. The worm (which is the driving member) is usually of a cylindrical form having threads of the same shape as that of an involute rack. The threads of the worm may be left handed or right handed and single or multiple threads. The worm wheel or gear (which is the driven member) is similar to a helical gear with a face curved to conform to the shape of the worm. The worm is generally made of steel while the worm gear is made of bronze or cast iron for light service.

The worm gearing is classified as non-interchangeable, because a worm wheel cut with a hob of one diameter will not operate satisfactorily with a worm of different diameter, even if the thread pitch is same.

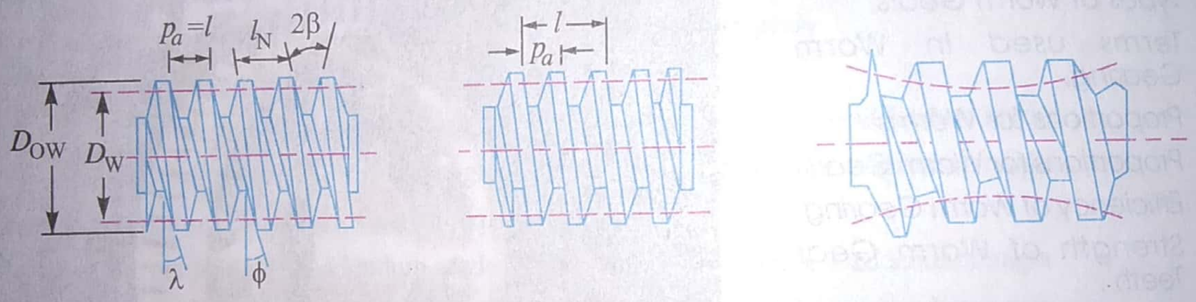
31.2 Types of Worms

The following are the two types of worms :

1. Cylindrical or straight worm, and
2. Cone or double enveloping worm.

The ***cylindrical*** or ***straight worm***, as shown in Fig. 31.1 (*a*), is most commonly used. The shape of the thread is involute helicoid of pressure angle 14 ½° for single and double threaded worms and 20° for triple and quadruple threaded worms. The worm threads are cut by a straight sided milling cutter having its diameter not less than the outside diameter of worm or greater than 1.25 times the outside diameter of worm.

The ***cone*** or ***double enveloping worm***, as shown in Fig. 31.1 (*b*), is used to some extent, but it requires extremely accurate alignment.



Single threaded. Double threaded.

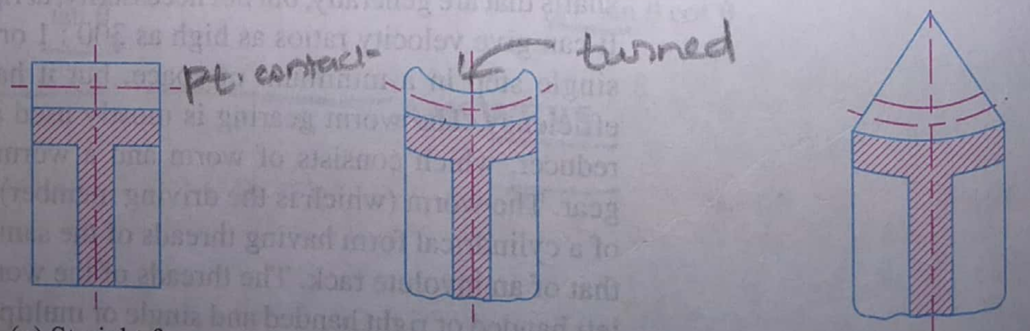
(*a*) Cylindrical or straight worm. (*b*) Cone or double enveloping worm.

Fig. 31.1. Types of worms.

31.3 Types of Worm Gears

The following three types of worm gears are important from the subject point of view :

1. Straight face worm gear, as shown in Fig. 31.2 (*a*),
2. Hobbed straight face worm gear, as shown in Fig. 31.2 (*b*), and
3. Concave face worm gear, as shown in Fig. 31.2 (*c*).



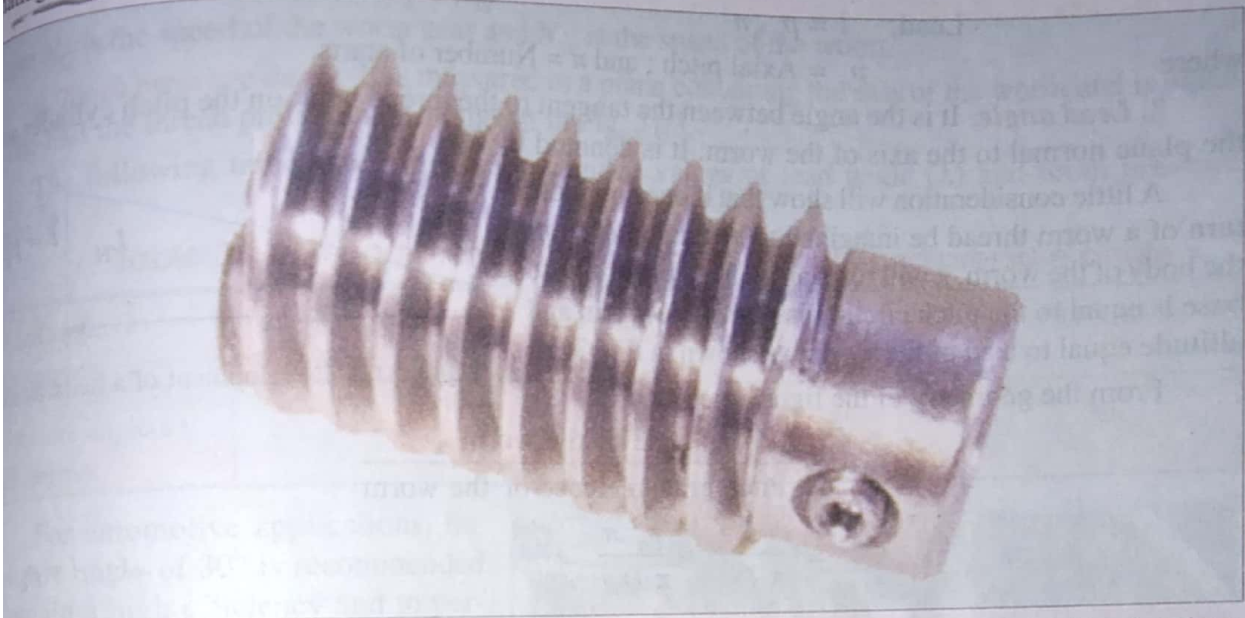
(*a*) Straight face. (*b*) Hobbed straight face. (*c*) Concave face.

Fig. 31.2. Types of worms gears.

The ***straight face worm gear*** is like a helical gear in which the straight teeth are cut with a form cutter. Since it has only point contact with the worm thread, therefore it is used for light service.

The ***hobbed straight face worm gear*** is also used for light service but its teeth are cut with a hob, after which the outer surface is turned.

The *concave face worm gear* is the accepted standard form and is used for all heavy service and eneral industrial uses. The teeth of this gear are cut with a hob of the same pitch diameter as the ating worm to increase the contact area.



Vorm gear is used mostly where the power source operates at a high speed and output is at a slow speed with high torque. It is also used in some cars and trucks.

1.4 Terms used in Worm Gearing

The worm and worm gear in mesh is shown in Fig. 31.3.

The following terms, in connection with the worm gearing, are important from the subject point view :

1. *Axial pitch.* It is also known as ***linear pitch*** of a worm. It is the distance measured axially e. parallel to the axis of worm) from a point on one thread to the corresponding point on the jacent thread on the worm, as shown in Fig. 31.3. It may be noted that the axial pitch (p_a) of a worm equal to the circular pitch (p_c) of the mating worm gear, when the shafts are at right angles.

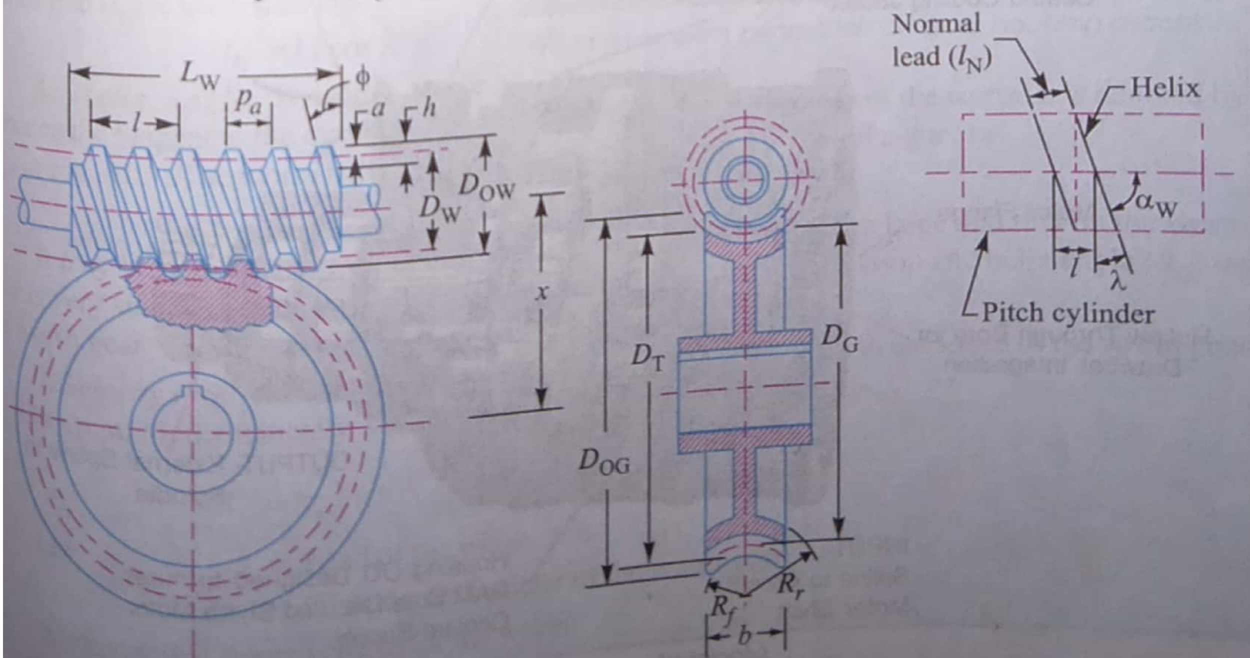


Fig. 31.3 . Worm and Worm gear.

2. *Lead.* It is the linear distance through which a point on a thread moves ahead in one revolution of the worm. For single start threads, lead is equal to the axial pitch, but for multiple start threads, lead is equal to the product of axial pitch and number of starts. Mathematically,

$$\text{Lead,} \quad l = p_a \cdot n$$

where p_a = Axial pitch ; and n = Number of starts.

3. *Lead angle.* It is the angle between the tangent to the thread helix on the pitch cylinder and the plane normal to the axis of the worm. It is denoted by λ.

A little consideration will show that if one complete turn of a worm thread be imagined to be unwound from the body of the worm, it will form an inclined plane whose base is equal to the pitch circumference of the worm and altitude equal to lead of the worm, as shown in Fig. 31.4.

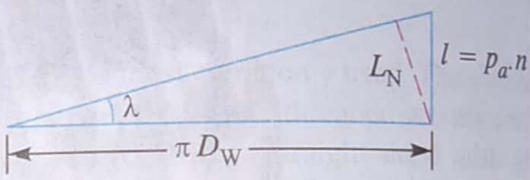


Fig. 31.4. Development of a helix thread.

From the geometry of the figure, we find that

$$\tan \lambda = \frac{\text{Lead of the worm}}{\text{Pitch circumference of the worm}}$$

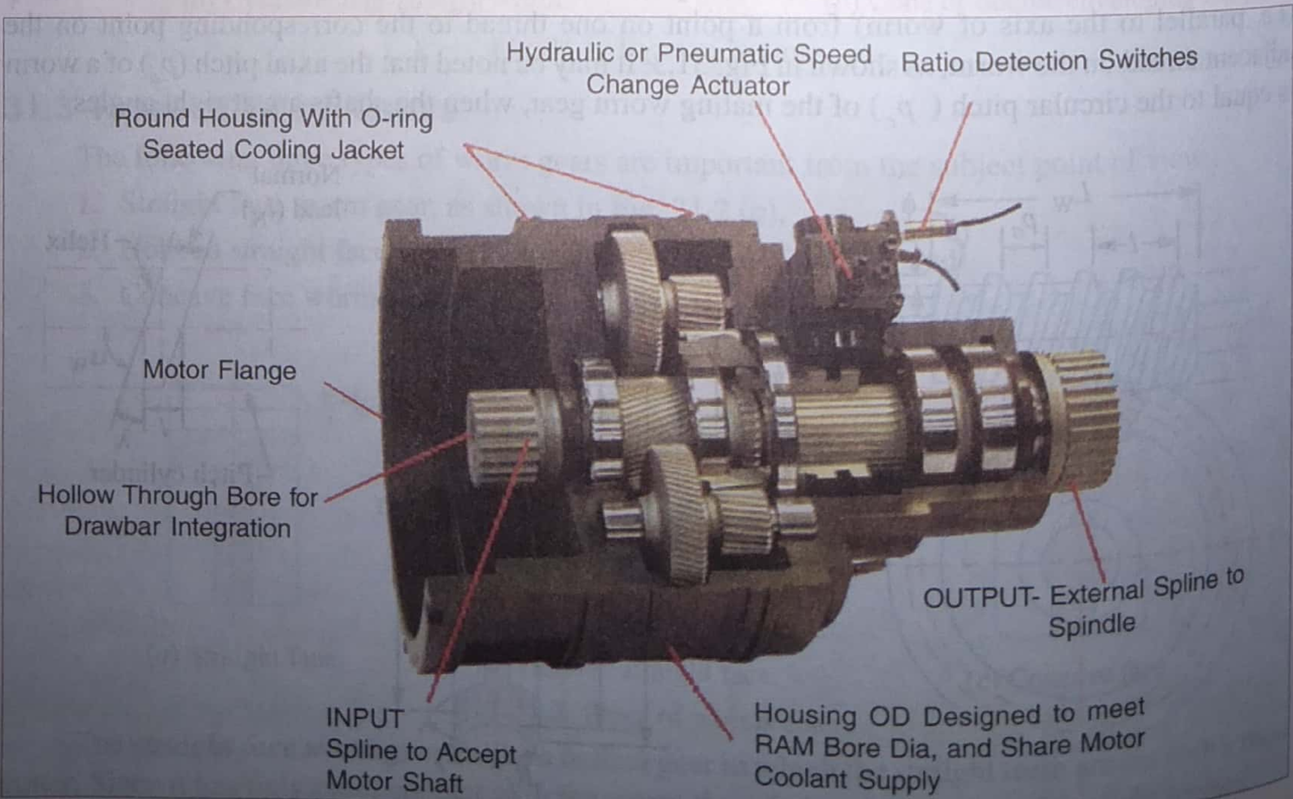
$$= \frac{l}{\pi D_W} = \frac{p_a . n}{\pi D_W} \qquad ...(\because l = p_a . n)$$

$$= \frac{p_c . n}{\pi D_W} = \frac{\pi m . n}{\pi D_W} = \frac{m . n}{D_W} \qquad ...(\because p_a = p_c \text{ ; and } p_c = \pi m)$$

where m = Module, and

D_W = Pitch circle diameter of worm.

The lead angle (λ) may vary from 9° to 45°. It has been shown by F.A. Halsey that a lead angle less than 9° results in rapid wear and the safe value of λ is 12½°.



Model of sun and planet gears.

For a compact design, the lead angle may be determined by the following relation, *i.e.*

$$\tan \lambda = \left(\frac{N_G}{N_W}\right)^{1/3},$$

where N_G is the speed of the worm gear and N_W is the speed of the worm.

4. ***Tooth pressure angle.*** It is measured in a plane containing the axis of the worm and is equal to one-half the thread profile angle as shown in Fig. 31.3.

The following table shows the recommended values of lead angle (λ) and tooth pressure angle (ϕ).

Table 31.1. Recommended values of lead angle and pressure angle.

Lead angle (λ) in degrees	0 – 16	16 – 25	25 – 35	35 – 45
Pressure angle(ϕ) in degrees	14½	20	25	30

For automotive applications, the pressure angle of 30° is recommended to obtain a high efficiency and to permit overhauling.

Worm gear teeth generation on gear hobbing machine.

5. ***Normal pitch.*** It is the distance measured along the normal to the threads between two corresponding points on two adjacent threads of the worm. Mathematically,

$$\text{Normal pitch, } p_N = p_a . \cos \lambda$$

Note. The term normal pitch is used for a worm having single start threads. In case of a worm having multiple start threads, the term normal lead (l_N) is used, such that

$$l_N = l \, . \cos \lambda$$

6. ***Helix angle.*** It is the angle between the tangent to the thread helix on the pitch cylinder and the axis of the worm. It is denoted by α_W, in Fig. 31.3. The worm helix angle is the complement of worm lead angle, *i.e.*

$$\alpha_W + \lambda = 90°$$

It may be noted that the helix angle on the worm is generally quite large and that on the worm gear is very small. Thus, it is usual to specify the lead angle (λ) on the worm and helix angle (α_G) on the worm gear. These two angles are equal for a 90° shaft angle.

7. ***Velocity ratio.*** It is the ratio of the speed of worm (N_W) in r.p.m. to the speed of the worm gear (N_G) in r.p.m. Mathematically, velocity ratio,

$$V.R. = \frac{N_W}{N_G}$$

Let l = Lead of the worm, and

D_G = Pitch circle diameter of the worm gear.

We know that linear velocity of the worm,

$$v_W = \frac{l . N_W}{60}$$

and linear velocity of the worm gear,

$$v_G = \frac{\pi\, D_G\, N_G}{60}$$

Since the linear velocity of the worm and worm gear are equal, therefore

$$\frac{l\,.N_W}{60} = \frac{\pi\, D_G\,.N_G}{60} \quad \text{or} \quad \frac{N_W}{N_G} = \frac{\pi\, D_G}{l}$$

We know that pitch circle diameter of the worm gear,

$$D_G = m\,.\,T_G$$

where m is the module and T_G is the number of teeth on the worm gear.

$$\therefore \qquad V.R. = \frac{N_W}{N_G} = \frac{\pi\, D_G}{l} = \frac{\pi\, m.T_G}{l}$$

$$= \frac{p_c\,.T_G}{l} = \frac{p_a\,.T_G}{p_a\,.n} = \frac{T_G}{n} \qquad \text{... } (\because p_c = \pi\, m = p_a;\ \text{and } l = p_a\,.\,n)$$

where $\qquad n$ = Number of starts of the worm.

From above, we see that velocity ratio may also be defined as the ratio of number of teeth on the worm gear to the number of starts of the worm.

The following table shows the number of starts to be used on the worm for the different velocity ratios :

Table 31.2. Number of starts to be used on the worm for different velocity ratios.

Velocity ratio (*V.R.*)	36 and above	12 to 36	8 to 12	6 to 12	4 to 10
Number of starts or threads on the worm ($n = T_w$)	Single	Double	Triple	Quadruple	Sextuple

31.5 Proportions for Worms

The following table shows the various porportions for worms in terms of the axial or circular pitch (p_c) in mm.

Table 31.3. Proportions for worm.

S. No.	*Particulars*	*Single and double threaded worms*	*Triple and quadruple threaded worms*
1.	Normal pressure angle (φ)	14½°	20°
2.	Pitch circle diameter for worms integral with the shaft	$2.35\,p_c + 10$ mm	$2.35\,p_c + 10$ mm
3.	Pitch circle diameter for worms bored to fit over the shaft	$2.4\,p_c + 28$ mm	$2.4\,p_c + 28$ mm
4.	Maximum bore for shaft	$p_c + 13.5$ mm	$p_c + 13.5$ mm
5.	Hub diameter	$1.66\,p_c + 25$ mm	$1.726\,p_c + 25$ mm
6.	Face length (L_W)	$p_c\,(4.5 + 0.02\,T_W)$	$p_c\,(4.5 + 0.02\,T_W)$
7.	Depth of tooth (h)	$0.686\,p_c$	$0.623\,p_c$
8.	Addendum (a)	$0.318\,p_c$	$0.286\,p_c$

Notes: 1. The pitch circle diameter of the worm (D_W) in terms of the centre distance between the shafts (x) may be taken as follows :

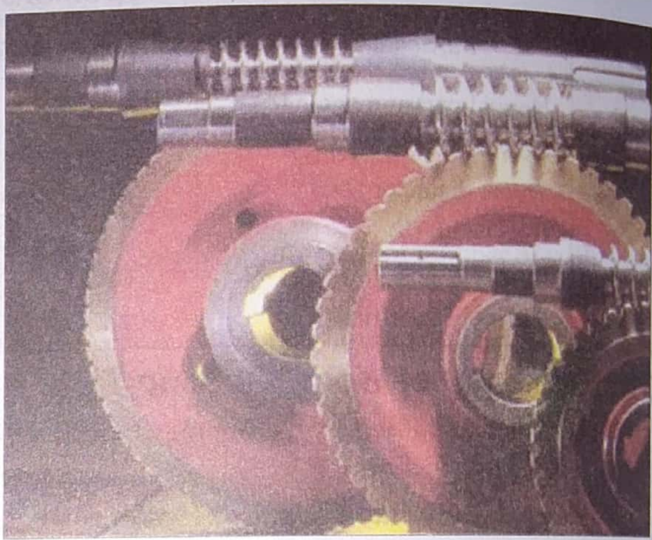
$$D_W = \frac{(x)^{0.875}}{1.416} \qquad \text{... (when } x \text{ is in mm)}$$

The coefficient of friction varies with the speed, reaching a minimum value of 0.015 at a rubbing speed $\left(v_r = \dfrac{\pi\, D_W \,.\, N_W}{\cos \lambda}\right)$ between 100 and 165 m/min. For a speed below 10 m/min, take $\mu = 0.015$. The following empirical relations may be used to find the value of μ, *i.e.*

$$\mu = \frac{0.275}{(v_r)^{0.25}}, \text{ for rubbing speeds between 12 and 180 m/min}$$

$$= 0.025 + \frac{v_r}{18000} \text{ for rubbing speed more than 180 m/min}$$

Note : If the efficiency of worm gearing is less than 50%, then the worm gearing is said to be ***self locking,*** *i.e.* it cannot be driven by applying a torque to the wheel. This property of self locking is desirable in some applications such as hoisting machinery.


Hardened and ground worm shaft and worm wheel pair

Example 31.1. *A triple threaded worm has teeth of 6 mm module and pitch circle diameter of 50 mm. If the worm gear has 30 teeth of 14½° and the coefficient of friction of the worm gearing is 0.05, find 1. the lead angle of the worm, 2. velocity ratio, 3. centre distance, and 4. efficiency of the worm gearing.*

Solution. Given : $n = 3$; $m = 6$; $D_W = 50$ mm ; $T_G = 30$; $\phi = 14.5°$; $\mu = 0.05$.

1. ***Lead angle of the worm***

Let λ = Lead angle of the worm.

We know that $\tan \lambda = \dfrac{m.n}{D_W} = \dfrac{6 \times 3}{50} = 0.36$

$\therefore$ $\lambda = \tan^{-1} (0.36) = 19.8°$ **Ans.**

2. ***Velocity ratio***

We know that velocity ratio,

$$V.R. = T_G / n = 30 / 3 = 10 \textbf{ Ans.}$$

3. ***Centre distance***

We know that pitch circle diameter of the worm gear

$$D_G = m.T_G = 6 \times 30 = 180 \text{ mm}$$

$\therefore$ Centre distance,

$$x = \frac{D_W + D_G}{2} = \frac{50 + 180}{2} = 115 \text{ mm } \textbf{Ans.}$$

4. ***Efficiency of the worm gearing***

We know that efficiency of the worm gearing.

$$\eta = \frac{\tan \lambda\,(\cos \phi - \mu \tan \lambda)}{\cos \phi \,.\, \tan \lambda + \mu}$$

$$= \frac{\tan 19.8°\,(\cos 14.5° - 0.05 \times \tan 19.8°)}{\cos 14.5° \times \tan 19.8° + 0.05}$$

$$= \frac{0.36\,(0.9681 - 0.05 \times 0.36)}{0.9681 \times 0.36 + 0.05} = \frac{0.342}{0.3985} = 0.858 \text{ or } 85.8\% \textbf{ Ans.}$$

CHAPTER – 9

WORM – GEAR DRIVE

9.1 BASIC CONCEPTS :

When a speed reducer is required to have a very high velocity ratio of about 100 or even more, sometimes upto 500, then, by two ways, this requirement may be fulfilled. One is by using spur, helical and bevel gears individually or in combined form in multi-stage arrangement. For example, if the required gear ratio is 100, then, this gear ratio may be obtained as $i = 5 \times 5 \times 4 = 100$ i.e., in three steps, because the maximum speed reduction for the above gear-drive in a single-step is about 6. Sometimes by using another type of gear-drive known as worm-gear drive, this much amount of speed ratio may be obtained in a single-step itself.

In the worm gear drive, the driving member is similar to an Archimedian screw and the driven member is similar to a helical gear with curved teeth and also the pitch surface of this helical gear is slightly concaved (i.e., shallow type pitch surface). In this drive, the driving member is known as worm in stead of calling as pinion and the driven member is known as worm-gear or worm wheel and the power is transmitted from worm to worm-wheel by sliding contact in contrast with spur, helical or bevel gears, where the power is transmitted by rolling contact.

The worm gear drive is employed to transmit power between non-parallel and non-interesting shafts whose axes are usually at 90°. Since they are having sliding contact, their operation is smooth and noiseless, but their efficiency is lower because of power loss due to friction caused by sliding. This drive is very compact for very high velocity ratio. Some of the drawbacks of this drive are such that the maximum power that can be transmitted by the drive is around 100 kW and usually this drive's elements may be kept inside the box, having full of oil in order to reduce the heat produced by the sliding friction. The worm may be cut usually with a single thread, (or) multiple threads, sometimes represented as single start or multiple starts.

Adv

$$\text{Velocity ratio, } i = \frac{\text{Number of teeth on gear}}{\text{Number of threads (or starts) on worm}}. = \frac{100}{1}$$

100 : 1

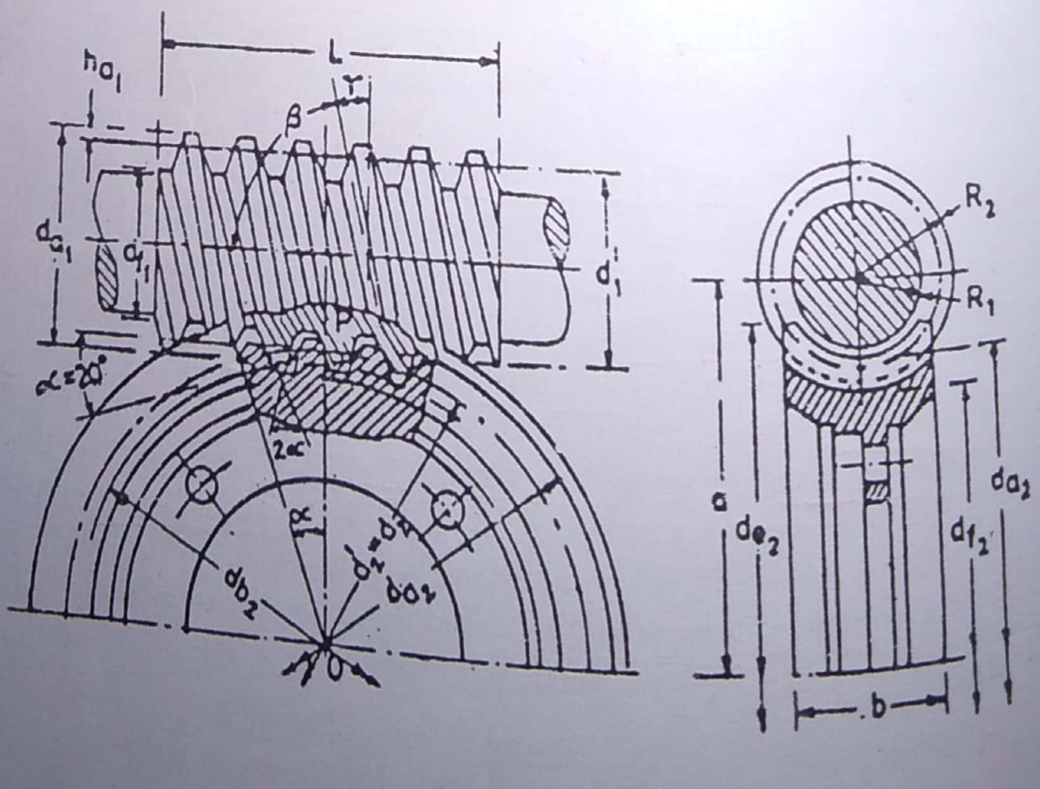
9.2 MATERIALS USED :

Since the worm is operated with worm-wheel by the sliding contact, the materials should have low coefficient of friction in order to reduce loss due to friction. This is ensured by combining heterogeneous materials with high quality finish of the contact surfaces. Usually carbon or alloy steels may be used to make worm and Gray cast-iron or bronze may be selected for making worm wheels. Among all the materials, "hardened steel and phospher bronze combination" has the lowest coefficient of friction and hence this material - combination may be preferred.

The helix angle, here known as lead angle, of worm-wheel should be kept within 30° so as to make the drive irreversible. If the lead angle is very high of about 70° to 80°, then, the worm gear drive may be operated as reversible drive. For example: Hand blower. For transmitting very high power of more than 500 kW at a very high velocity ratio, spur gear drive with multi-stage speed reduction is preferred to worm gear-drive in order to avoid more heat produced during operation in worm gear drive and also its inefficiency to transmit such a high power.

9.3 DESIGN PARAMETERS :

Since the worm gear drive transmite power through sliding contact, we should consider the friction effect when designing. Let us consider a worm gear drive as shown in figure 9.1. for which the essential parameters regarding the design point of view are given below.



Worm-gear drive

Fig. 9.1

1) a = Centre distance, – the distance between the axes of worm and worm-wheel.

2) Z_1 = Number of starts or threads on worm, usually from 1 to 4

3) Z_2 = Number of teeth on worm-wheel, which may be selected from 25 to 85. Usually for higher power, Z_2 may be taken as 60 to 80 and for lower power, Z_2 may be taken as 25 to 50.

4) q = Diameter factor which is the ratio between the pitch diameter of worm d_1, and the axial module, m_x, i.e., $q = \dfrac{d_1}{m_x}$

5) $[\sigma_c]$ = Design surface stress (Table 9.1) (PSG 8.45)

6) $[\sigma_b]$ = Design bending stress (Table 9.2) (PSG 8.45)

7) $[M_t]$ = Design torque $= M_t \cdot k \cdot k_d$, where M_t is the nominal torque obtained as $M_t = \dfrac{60\,P}{2\,\pi\,n_2}$ (or) $\dfrac{60\,P}{2\,\pi\left(\dfrac{n_1}{i}\right)}$

K = Load concentration factor = 1 for constant loading.

k_d = Dynamic load factor = 1 for $V_2 < 3$ m/s where V_2 is the velocity of worm wheel.

8) v = Helix angle or Lead angle i.e., $\tan v = \left(\dfrac{Z_1}{q}\right)$ (Table ~~9.4~~ 34 (PSG 8.45))

9) i = Gear ratio or velocity ratio $= \dfrac{Z_2}{Z_1} = \dfrac{n_1}{n_2}$.

10) η = Efficiency of worm - gear drive (Table ~~9.5~~ 37) (PSG 8.46)

9.4 FORMULAS USED :

The root formulas required for the design of worm-gear drive are as follows.

i) The minimum centre distance based on surface contact stress has been derived for hardened steel worm and phospher bronze wheel combination as

$$a \geq \left(\frac{Z_2}{q} + 1\right) \sqrt[3]{\left[\frac{540}{\frac{Z_2}{q}[\sigma_c]}\right]^2 [M_t]}$$

Note : For steel worm and cast-iron wheel, the number 540 in the above formula is replaced by 978.

ii) The minimum axial module based on bending stress has been derived as

$$m_x \geq 1.24 \sqrt[3]{\frac{[M_t]}{[\sigma_b]\, q\, Z_2\, y_v}}$$

where y_v– Form factor corresponding to the virtual number of teeth of wheel, $Z_{2v} = \frac{Z_2}{\cos^3 v}$

9.5 DESIGN PROCEDURE :

1 From the given problem, note down the amount of power to be transmitted, speed ratio, worm speed, materials required etc. Usually the steel for worm and bronze for wheel are preferred.

2. Estimate the minimum centre distance, a, based on surface compressive strength as

$$a \geq \left\{\frac{Z_2}{q} + 1\right\} \sqrt[3]{\left[\frac{540}{\frac{Z_2}{q}[\sigma_c]}\right]^2 [M_t]}$$ for steel worm and bronze wheel combination

where $[\sigma_c]$ = Design surface compressive stress which depends on sliding velocity, V_s (from Table ~~8.41~~) (PSG 8.45)

35 8.44

PSG 8.44

(Initially V_s is assumed 3 m/s)

q = Diameter factor $\frac{d_1}{m_x} = 11$ (Assume initially)

ok

Z_2 = Number of teeth on worm wheel

= $i\,Z_1$ where Z_1 = number of starts on worm and i = Velocity ratio

$[M_t]$ = Design torque $= M_t \cdot k \cdot k_d$,

Initially k.kd may be assumed as 1.

3. Determine the minimum axial module, m_x, based on bending stress as

$$m_x \geq 1.24 \sqrt[3]{\frac{[M_t]}{[\sigma_b]\, q\, Z_2\, y_v}}$$

ok 35 8.45 ok

where $[\sigma_b]$ = Design bending stress (From Table 9.2) (PSG 8.45)

$$y_v = \text{Form factor corresponding } Z_{v2} = \frac{Z_2}{\cos^3 \nu}$$

and ν is the helix angle or lead angle of worm wheel which can be obtained from the relation as

34 ok ok

$\nu = \tan^{-1} \dfrac{Z_1}{q}$ or from table 9.4 (PSG 8.45)

1 ok

4. Standardise this module using the table 6.16 (PSG 8.2)

5. Find out the diameter factor, q, and then, the centre distance using the relation as

$$a = 0.5\, m_x\, (q + Z_2).$$

6. Estimate the actual sliding velocity using

$$V_s = \frac{\pi\, d_1\, n_1}{60 \times 1000 \times \cos \nu} \text{ m/s}$$

and note the corresponding design surface from the table 9.1

7. Calculate the induced surface compressive stress and bending stress as below and check them with their permissible (i.e., design) values.

$$\text{i.e., } \sigma_c = \left[\frac{540}{\left(\frac{Z_2}{q}\right)}\right] \sqrt{\left[\frac{\frac{Z_2}{q}+1}{a}\right]^3 [M_t]} \le [\sigma_c]$$

$$\text{and } \sigma_b = \frac{1.9\,[M_t]}{m_x^3\, q\, Z_2\, y_v}$$

8. Determine the length of worm as

39 ok

$L \ge (11 + 0.06\, Z_2)\, m_x$ for $Z_1 = 1$ to 2 (Table 9.8) (PSG 8.48)

and $L \ge (12.5 + 0.09\, Z_2)\, m_x$ for $Z_1 = 3$ to 4

For ground worm, the length L is increased by some amount, to become a new length, L_1

i.e., $L_1 = L + 25$ mm for $m_x < 10$ mm

$= L + 35$ to 40 mm for $m_x = 10$ to 16 mm

$= L + 50$ mm for $m_x > 16$ mm

9. Decide the number of teeth on worm λ as

$\lambda = \dfrac{L_1}{\pi\, m_x}$ and correct it to full number and then findout the actual length (L_2) of worm for that corrected number of teeth as

$L_2 = \lambda\, \pi\, m_x$

10. Determine the face width of worm-wheel by an empirical relation given in the table 9.6 (PSG 8.48)

ok 38 ok

Given in reference

11. Compute the parameters of worm and worm wheel such as their reference (or pitch) diameters, tip diameters, root diameters etc. using the relations given in table 9.3 (PSG 8.43)

relations

35

ok ok

12. Calculate the efficiency of the drive as

$$\eta = \frac{\tan \nu}{\tan(\nu + \rho)} \text{ where } \rho = \tan^{-1}\mu \text{ (Refer figure 9.2)}$$

Table 9.7

Pg 8.46

ν = Lead Angle

ρ = Friction Angle in Degree

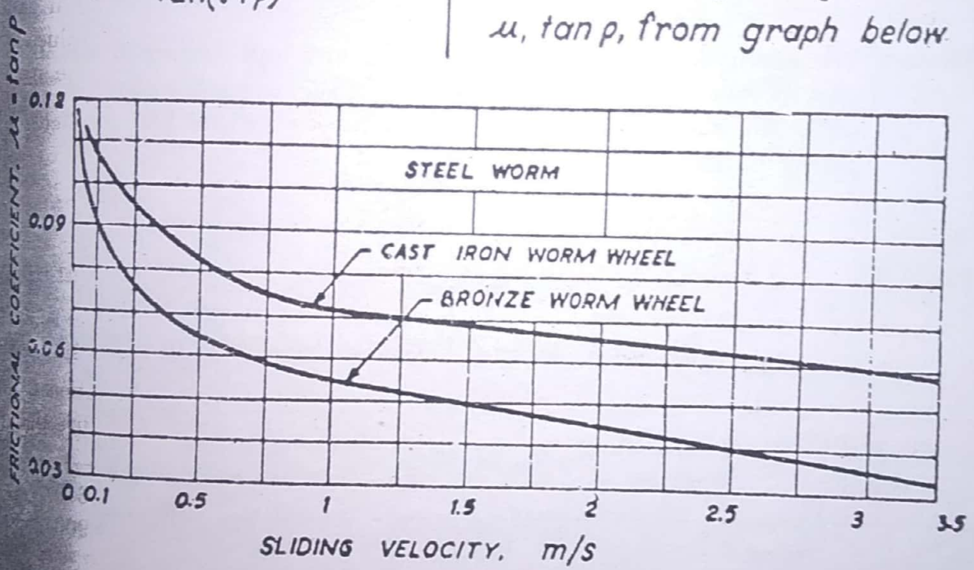
EFFICIENCY OF WORM GEAR DRIVE (when worm is driving)

$$\eta = \frac{\tan \gamma}{\tan(\gamma + \rho)}$$

γ, lead angle, deg.

ρ, friction angle deg.

μ, tan ρ, from graph below



where ρ

Fig. 9.2

where ρ = Friction angle in degrees and μ = coefficient of friction.

13. Draw a neat sketch of worm-gar drive.

13)

3M-101

Table 9.1

Design Surface Stress*, $[\sigma_c]$, kgf/cm^2

Material		Sliding Velocity, V_s, m/s					
Worm	Wheel	0.25	0.5	1	2	3	4
Steel	Cast Iron	1700-1400	1200	1000	700	–	–
	Bronze	1900	1850	1760	1680	1590	14[illegible]

*Note : The above values are for accurately cut and well lubricated worm pair. If these two conditions are not satisfied, reduce the above value 30%, The above values need not modified for fatigue life, as the failure in worm gearing is due to seizure.

Table 9.2

Design Bending Stress, $[\sigma_b]$, kgf/cm^2

Material of wheel	Method of casting of wheel	σ_u, kgf/mm^2	Rotation in one direction only	Rotation both direc
Bronze	Sand	$\sigma_u > 39$	780	640
	Chill		1100	900
	Sand	$\sigma_u < 39$	500	350
	Chill		550	400
	Centrifugally cast		600	470
Cast Iron	Grade 25	25	300	200
	Grade 35	35	400	27[illegible]

Table 9.3

BASIC DIMENSIONS OF WORM GEAR PAIR

Nomenclature	Symbol	Formulae	Nomenclature	Symbol	Formulae
Centre distance	a	$a = 0.5\ m_x (q + Z_2 + 2x)$	WORM		
Face width of wheel	b		Reference diameter	d_1	$d_1 = q\ m_x$
Length of work	L		Tip diameter	d_{a1}	$d_{a1} = d_1 + 2\ f_0\ m_x$
			Root diameter	d_{f1}	$d_{f1} = d_1 - 2\ f_0\ m_x - 2\ c$
Axial module	m_x	$m_x = \frac{2a}{q + z2 + 2x}$	Pitch diameter	d_1'	$d_1' = m_x (q + 2x)$
Number of starts	Z_1	Determined by calculation	WORM WHEEL (In axial section)		
Number of teeth on wheel	Z_2	$Z_2 = i\ Z_1$	Reference diameter	d_2	$d_2 = z_2\ m_x$
Gear ratio	i	$i = \frac{Z_2}{Z_1}$	Tip diameter	d_{a2}	$d_{a2} = (Z_2 + 2\ f_0 + 2x)\ m_x$
Diameter factor	q	$q = \frac{d_1}{m_x}$	Root diameter	d_{f2}	$d_{f2} = (z_2 - 2\ f_0)\ m_x - 2\ c$
			Pitch diameter	d_2'	$d_2' = d_2$
Height factor	f_0	Normally $f_0 = 1$	WORM WHEEL THROAT		
Pitch of the helix, lead	p_z	$p_z = Z_1\ p_x$	Axial pitch	p_x	$p_x = \pi\ m_x$
Pressure Angle in axial section (Archimedean worm)	∞	$\infty = 20°$	Thorat tip radius	R_1	$R_1 = \frac{d_1}{2} f_0\ m_x$
			Throat root radius	R_2	$R_2 = \frac{d_1}{2} f_0\ m_x + c$
			WORM		
			Tip relief radius	r1	$r1 = 0.1\ m_x$
Lead angle one the reference cylinder	v	$\tan v = \frac{z_1}{q}$	Root relief radius	r2	$r2 = 0.2\ m_x$
Lead angle on the Pitch cylinder	v	$\tan v = \frac{Z_1}{(q + 2x)}$	Nominal tooth thickness on reference diameter in axial section	S	$S = \frac{\pi\ m_x}{2}$
Bottom clearance	c	$c = 0.2$ to $0.3\ m_x$			
Addendum modification coefficient	x	$x = \frac{a}{m_x} - 0.5\ (q + Z_2)$	Nominal tooth thickness on reference diameter in normal section	S_n	$S_n = \frac{\pi}{2} m_x \cos y$

Table. 35 Pg no. 8.43

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Table 9.4

Lead Angle, γ

No. of starts Z	Diameter Factor, q					
	13	12	11	10	9	8
1.	4° 23′ 55″	4° 45′ 49″	5° 11′ 40″	5° 42′ 38″	6° 20′ 25″	7° 07′ 30″
2.	8° 44′ 46″	9° 27′ 44″	10° 18′ 17″	11° 18′ 36″	12° 31′ 44″	14° 02′ 1
3.	12° 59′ 41″	14° 02′ 10″	15° 15′ 18″	16° 41′ 57″	18° 26′ 06″	20° 33′ 2
4.	17° 06′ 10″	18° 26′ 06″	19° 58′ 59″	21° 48′ 05″	23° 57′ 45″	26° 33′ 5

Table 9.5: Approximate Efficiencies in worm Gear Drive

For Z = 1, $\eta = 0.70$ to 0.75

= 2, $\eta = 0.70$ to 0.82

= 3 and 4, $\eta = 0.80$ to 0.92

Table 9.6: Face width of the wheel, b

Z	b
1	$0.75\ d_1$
2 or 3	$0.75\ d_1$
4	$0.67\ d_1$

d_1 = Pcd of worm

Table 9.7: Selection of number of teeth, Z_2 on the worm wheel**

m_x, mm	4	5	5	6	6	8	8	10	10	12	12	16	20
q → / a, mm ↓	11	10	12	9	11	8	11	8	11	8	11	9	8
100	39	30	28	–	–	–	–	–	–	–	–	–	–
120	49	38	36	31	29	–	–	–	–	–	–	–	–
150	64	50	48	41	39	29*	–	–	–	–	–	–	–
180	79	62	60	51	49	37	34	28	–	–	–	–	–
210	–	74	72	61	59	44*	41*	34	31	–	–	–	–
240	–	–	–	71	69	52	49	40	37	32	29	–	–
270	–	–	–	81	79	59*	56*	46	43	37	34	–	–
300	–	–	–	–	–	67	64	52	49	42	39	28*	–
360	–	–	–	–	–	82	79	64	61	52	49	36	28
420	–	–	–	–	–	–	–	76	73	62	59	43*	34
480	–	–	–	–	–	–	–	–	–	72	69	51	40
540	–	–	–	–	–	–	–	–	–	82	79	58*	46
600	–	–	–	–	–	–	–	–	–	–	–	66	52

**Z_2 could be ±2 teeth from values found from the table above except numbers marked with* where the deviation permissible is −1 or +2 teeth.

Table 9.8

Length of the worm, L

Correction factor x	Z_1 no. of starts: 1 or 2	Z_1 no. of starts: 3 or 4
0	$L \geq (11 + 0.06\ Z_2)\ m_x$	$L \geq (12.5 + 0.09\ Z_2)\ m_x$
−0.5	$L \geq (8 + 0.06\ z_2)\ m_x$	$L \geq (9.5 + 0.09\ z_2)\ m_x$
−1	$L \geq (10.5 + Z_2)\ m_x$	$S \geq (10.5 + Z_2)\ m_x$
+0.5	$L \geq (11 + 0.1 Z_2)\ m_x$	$L \geq (12.5 + 0.1\ Z_2)\ m_x$
1	$L \geq (12 + 0.1\ Z_2)\ m_x$	$L \geq (13 + 0.1\ Z_2)\ m_x$

3M-105

4M-15

9.6. PROBLEMS :

Problem 9.1

The input to worm gear shaft is 18 kW and 600 rpm. Speed ratio is 20. The worm is to be of hardened steel and the wheel is made of chilled phospher bronze. Considering wear and strength, design worm and worm wheel.

Solution :

load = π × m × no of start

Power to be transmitted	=	18 kW
Speed of worm	=	600 rpm
Speed ratio	=	20
Material for worm	=	Hardened steel
Material for wheel	=	Phospher Bronze (Chilled)

Minimum centre distance between worm and worm wheel based on surface stress is given by,

σ_c = Design surface stress

$$a \geq \left(\frac{Z_2}{q}+1\right)\sqrt[3]{\left\{\frac{540}{\frac{Z_2}{q}[\sigma_c]}\right\}^2 [M_t]}$$

Pg no 8.44 PSG DD

$\frac{60}{3} = 20$

$$M_t = 97420 \times \frac{kW}{n_1} i\,\eta \qquad \text{(where } n_1 = \text{ speed of worm} = 600 \text{ rpm)}$$

$i = \frac{Z_2}{Z_1} = 20$, Take q = 11 initially

q = Diameter factor

Z_1 = No. of starts on the worm = 3

$\therefore \eta = 0.86$ (Assume) (Table ~~9.5~~) 37 Pg 8.46

$q = \frac{d_1}{m_x}$

$\therefore Z_2 = 3 \times 20 = 60$

$d_1 = q\,m$

$$M_t = 97420 \times \frac{18}{600} \times 20 \times 0.86 = 50270 \text{ kgf-cm}$$

$$[M_t] = M_t \cdot k \cdot k_d$$

$$= 50270 \times 1 \times 1$$

$$= 50270 \text{ kgf-cm}$$

$k = 1$ ∵ load is almost constant

$k_d = 1$ for assumed sliding velocity V_s of

$V_s = 3$ m/sec

$V_s = \frac{\pi d n}{60 \times 1000 \times \cos\gamma}$

$= 3.0$

$d_1 \times 600$

15° 18' 9.648

Material Selection :

For worm (steel) and wheel (phospher Bronze)

$[\sigma_c] = 1590$ kgf/cm^2 Assuming $V_s = 3$ m/sec — Table 32, Pg no 8.45

$[\sigma_b] = 550$ kgf/cm^2 (Table ~~9.2~~ 33) (PSG8.45)

$$\text{Now } a \geq \left(\frac{60}{11}+1\right)\sqrt[3]{\left[\frac{540}{\left(\frac{60}{11}\right)(1590)}\right]^2 \times 50270}$$

≥ 37.4 cm

Axial Module based on beam strength

$$m_x \geq 1.24\sqrt[3]{\frac{[M_t]}{Z_2\, q \cdot y_v \cdot [\sigma_b]}}$$

y_v = Form factor for virtual no. of teeth Z_{v2}

$$Z_{v2} = \frac{Z_2}{\cos^3 v} \quad \text{where } v = \text{lead angle or helix angle}$$

$\tan \gamma = \frac{Z_1}{q}$

$$v = \tan^{-1}\left(\frac{Z_1}{q}\right) = \tan^{-1}\left(\frac{3}{11}\right) = 15° \, 15' \, 18''$$

$$= 15.255°$$

$$Z_{v2} = \frac{60}{\cos^3 15.255} = 66.8 = 67$$

$y_v = 0.493$ (Table ~~6.25~~ 18) (PSG8.18)

Against virtual no of teeth 67 & for Zero Correction for worm wheel

$[\sigma_b] = 550$ kgf/cm^2

$$m_x \geq 1.24\sqrt[3]{\frac{50270}{60 \times 11 \times 0.493 \times 550}}$$

≥ 0.82 cm ≥ 8.2 mm

Take $m_x = 10$ mm

3M-107

Now the centre distance

$a = 0.5\ m_x\ (q + Z_2 + 2\ x)$

$= 0.5 \times 10\ (11 + 60)$ (Assuming $x = 0$)

$= 355$ mm

(x = correcting Pg no 8.43 PSG DD Table 35)

Since this is less than the minimum centre distance (= 374 mm)

Let $m_x = 12$ mm

Now $a = 0.5 \times 12\ (11 + 60) = 426$ mm $= 42.6$ cm (O.K)

Sliding velocity $V_s = \dfrac{\pi\ d_1\ n_1}{60 \times 1000 \times \cos v}$

$d_1 = q\ m_x = 11 \times 12 = 132$ mm

Pcd (d_1) = $q\ m_x$

$v = 15.255°$

$$V_s = \frac{\pi \times 132 \times 600}{60 \times 1000 \times \cos 15.255°} = 4.29 \text{ m/sec}$$

Using another formula : (PSG 8.44)

$$V_s = \frac{m_x \cdot n}{19100}\sqrt{Z_1^2 + q^2}$$

$$= \frac{12 \times 600}{19100}\sqrt{3^2 + 11^2} = 4.29 \text{ m/sec}$$

Since adequate data are not available for surface strength for sliding beyond 4 m/sec it is assumed that $[\sigma_c] = 1490$ kgf/cm^2. (Table ~~81~~ 32) (PSG 8.[illegible])

Checking :

$$\sigma_c = \frac{540}{\left(\frac{Z_2}{q}\right)}\sqrt{\left\{\frac{\left(\frac{Z_2}{q}+1\right)}{a}\right\}^3 [M_t]}$$

Pg no : 8.44

$$= \frac{540}{\left(\frac{60}{11}\right)}\sqrt{\left\{\frac{\left(\frac{60}{11}+1\right)}{42.6}\right\}^3 50270}$$

$= 1300$ kgf/cm^2 $< [\sigma_c] = 1490$ kgf/cm^2

Table 35 pg 8-44

$$\sigma_b = \frac{1.9\,[M_t]}{m_x^3\, q\, Z_2\, y_v}$$

$$= \frac{1.9 \times 50270}{1.2^3 \times 11 \times 60 \times 0.493} = 170\ \text{kgf/cm}^2 < [\sigma_b] = 550\ \text{kgf/cm}^2$$

Our design is safe.

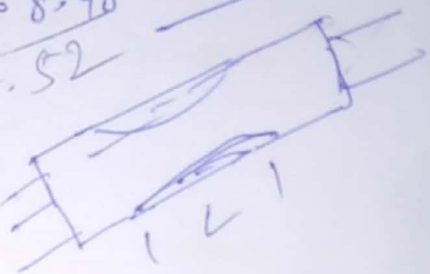
Length of worm $L \geq (12.5 + 0.09\, Z_2)\, m_x$

Pg no 8.48 & 8.52 PSG DD

$L \geq (12.5 + 0.09 \times 60)\ 12$

≥ 214.8 mm

≥ 215 mm



Pitch

NO

No. of teeth on worm $= \lambda = \dfrac{L}{\pi\, m_x}$

$$= \frac{215}{\pi \times 12} = 5.7$$

Let $\lambda = 6$

Lead = $\pi \times m_x \times$ No of start

Now the length of worm $= 6 \times \pi \times m_x$

$= 6 \times \pi \times 12 = 226$ mm

Breadth

Breadth of worm wheel $= 0.75\, d_1$

(Face width) $= 0.75 \times 132 = 99$ mm

$= 100$ mm (say)

Para

Parameters of worm : (Refer table ~~9.3~~ 35) (PSG 8.43)

Table 35 page no 8-43 PSG DD

Reference diameter $d_1 = q\, m_x = 132$ mm

Tip diameter $d\,a_1 = d_1 + 2\, f_0\, m_x$

Tip

$= 132 + (2 \times 1 \times 12) = 156$ mm

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Root diameter $df_1 = d_1 - 2\,f_0\,m_x - 2c$

$$= 132 - 2 \times 1 \times 12 - 2 \times 0.2 \times 12$$

$$= 132 - 2 \times 12\,(1 + 0.2)$$

$$= 132 - 28.8$$

$$= 103.2 \text{ mm}$$

Pitch diameter $d_1' = m_x\,(q + 2\,x)$

$$= 12\,(11 + 2 \times 0) \text{ (Assuming } x = 0)$$

$$= 132 \text{ mm}$$

Wheel :

Reference diameter $d_2 = Z_2\,m_x$

$$= 60 \times 12 = 720 \text{ mm}$$

Tip diameter $da_2 = (Z_2 + 2\,f_0 + 2\,x)\,m_x$

$$= (60 + 2)\,12 = 744 \text{ mm}$$

Root diameter $d\,f_2 = (Z_2 - 2\,f_0)\;m_x - 2c$

$$= (60 - 2)\,12 - (2 \times 0.2 \times 12)$$

$$= 691.2 \text{ mm}$$

Pitch diameter $= d_2' = d_2 = 720$ mm

Efficiency of worm gear drive :

$$\eta = \frac{\tan \nu}{\tan(\nu + \rho)}$$

Pg No 8.49 PSG DD

$\tan \rho = \mu =$ Friction coefficient

$= 0.03$ for sliding velocity $V_s = 4.29$ m/sec (Refer figure 9.2)

Pg 8.49

$\mu = \tan\varphi = 0.03$

$$\rho = \tan^{-1} 0.03 = 1.72°$$

$$\therefore \eta = \frac{\tan(15.255)}{\tan(15.255 + 1.72)} = 0.893 = 89.3\%$$

Specifications :

Sl. No.	Description	Worm	Wheel
1.	Material	Steel	Phosphor Bronze
2.	No. of teeth	6	60
3.	Module	12 mm	12 mm
4.	Reference Diameter	132 mm	720 mm
5.	Tip diameter	156 mm	744 mm
6.	Root diameter	103.2 mm	691.2 mm
7.	Length of worm	226 mm	–
8.	Face width of worm wheel	–	100 mm
9.	Centre distance	426 mm	
10.	Efficiency of drive	89.3 %	

9.7 SHORT QUESTIONS :

1. What is a worm-gear drive?

Worm gear-drive is a type of gear drive in which pinion is made as lead screw shape and the wheel is made as helical gear structure and it is employed for transmitting power between the shafts whose axes are at right angle to each other. Also this gear drive is used for high velocity ratio of about 100, sometimes upto 500.

2. What kind of contact occured between worm and wheel? How does this differ from other gears?

   In worm gear drive, sliding contact is occured between the worm and wheel whereas other gears such as spur, helical and bevel gears transmit power by rolling contact.

3. In which gear-drive, self-locking is available?

   Self-locking is available in worm-gear drive.

4. When do we use worm-gears?

   When we require to transmit power between nonparallel and non-intersecting shafts and very high velocity ratio of about 100, worm gears can be employed. Also worm-gears provide self-locking facility.

5. Write some applications of worm-gear drive.

   Worm gear drive find wide applications like hoisting equipments, milling machine indexing head, table fan, steering rod of automobile and so on.

6. Suggest one material for the following giving the reason.

   a) Worm. b) Worm-wheel. c) Bevel gears.

   a) Worm – Steel to transmit high torque.

   b) Worm wheel – Phospher bronze to get low coefficient of friction when the wheel is operated with worm.

   c) Bevel gears – Hardened steel to sustain heavy load and to transmit high power.

7. Define the following terms.

   a) Pitch. b) Lead.

   a) Pitch of worm is the distance, measured axially, from a point on one tooth to the corresponding point on an adjacent tooth.

   b) Lead of worm is defined as the distance, measured axially, between the corresponding points of adjacent teeth for the same helix. Based on the number of starts, lead can be varied from pitch. Usually,

   Lead = number of starts × pitch

8. State True or False.

a) The contact between worm and wheel is the sliding contact.

b) The materials used for worm gears should have high coefficient of friction.

Answers :

a) True

b) False.

9. What are the merits and demerits of worm-gear drive?

Merits :

1) Compact layout for very high velocity ratio of about 100.

2) Smooth and noiseless operation.

3) Self-locking facility is available.

Demerits :

1) Low efficiency.

2) More heat will be produced and hence this drive can be operated inside an oil reservoir or extra cooling fan is required in order to dissipate the heat from the drive.

3) Low power transmission.

9.8 EXERCISES :

1. What is a worm gear and where is it used?

2. What is meant by self-locking? At which gear it is available?

3. Why is worm gear operated inside an oil reservoir?

4. Write short notes on the following.

a) Lead and Pitch.

b) Diameter factor.

5. What is the significant of using heterogeneous materials

21

Chain Drives

21.1 Introduction

We have seen in previous chapters on belt and rope drives that slipping may occur. In order to avoid slipping, steel chains are used. The chains are made up of number of rigid links which are hinged together by pin joints in order to provide the necessary flexibility for wraping round the driving and driven wheels. These wheels have projecting teeth of special profile and fit into the corresponding recesses in the links of the chain as shown in Fig. 21.1. The toothed wheels are known as ****sprocket wheels or simply sprockets***. The sprockets and the chain are thus constrained to move together without slipping and ensures perfect velocity ratio.

* These wheels resemble to spur gears.

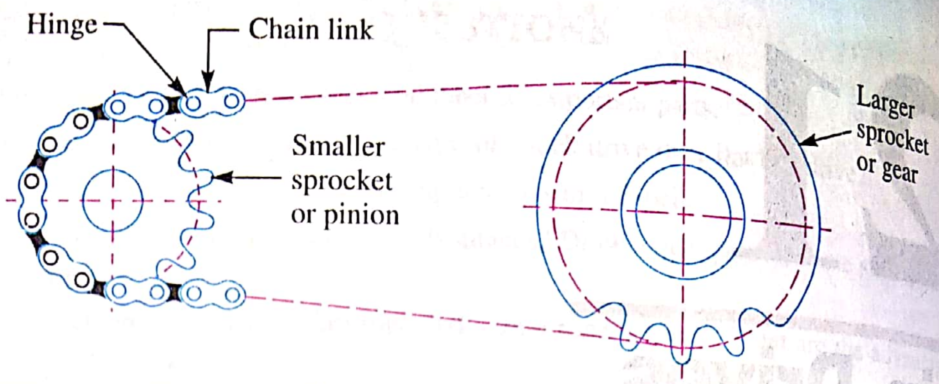


Fig. 21.1. Sprockets and chain.

The chains are mostly used to transmit motion and power from one shaft to another, when the centre distance between their shafts is short such as in bicycles, motor cycles, agricultural machinery, conveyors, rolling mills, road rollers etc. The chains may also be used for long centre distance of upto 8 metres. The chains are used for velocities up to 25 m / s and for power upto 110 kW. In some cases, higher power transmission is also possible.

21.2 Advantages and Disadvantages of Chain Drive over Belt or Rope Drive

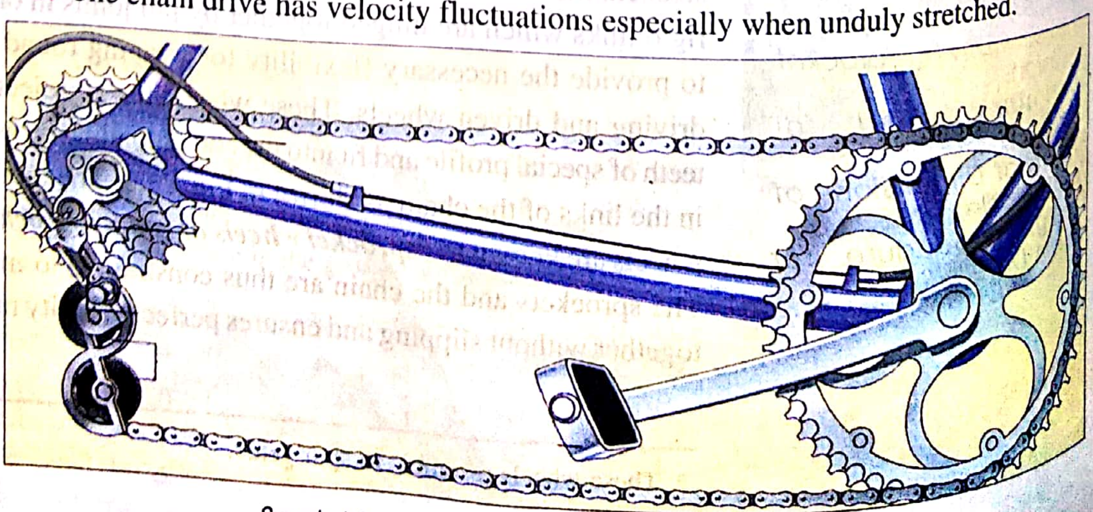
Following are the advantages and disadvantages of chain drive over belt or rope drive:

Advantages

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.
2. Since the chains are made of metal, therefore they occupy less space in width than a belt or rope drive.
3. It may be used for both long as well as short distances.
4. It gives a high transmission efficiency (upto 98 percent).
5. It gives less load on the shafts.
6. It has the ability to transmit motion to several shafts by one chain only.
7. It transmits more power than belts.
8. It permits high speed ratio of 8 to 10 in one step.
9. It can be operated under adverse temperature and atmospheric conditions.

Disadvantages

1. The production cost of chains is relatively high.
2. The chain drive needs accurate mounting and careful maintenance, particularly lubrication and slack adjustment.
3. The chain drive has velocity fluctuations especially when unduly stretched.



Sports bicycle gear and chain drive mechanism

21.3 Terms Used in Chain Drive

The following terms are frequently used in chain drive.

1. *Pitch of chain.* It is the distance between the hinge centre of a link and the corresponding hinge centre of the adjacent link, as shown in Fig. 21.2. It is usually denoted by p.

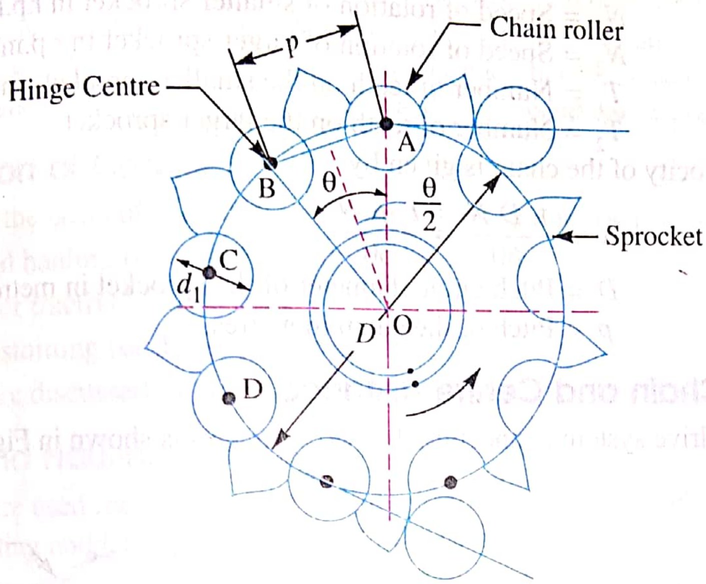


Fig. 21.2. Terms used in chain drive.

2. *Pitch circle diameter of chain sprocket.* It is the diameter of the circle on which the hinge centres of the chain lie, when the chain is wrapped round a sprocket as shown in Fig. 21.2. The points A, B, C, and D are the hinge centres of the chain and the circle drawn through these centres is called pitch circle and its diameter (D) is known as pitch circle diameter.

21.4 Relation Between Pitch and Pitch Circle Diameter

A chain wrapped round the sprocket is shown in Fig. 21.2. Since the links of the chain are rigid, therefore pitch of the chain does not lie on the arc of the pitch circle. The pitch length becomes a chord. Consider one pitch length AB of the chain subtending an angle θ at the centre of sprocket (or pitch circle),

Let

D = Diameter of the pitch circle, and

T = Number of teeth on the sprocket.

From Fig. 21.2, we find that pitch of the chain,

$$p = AB = 2\,AO\sin\left(\frac{\theta}{2}\right) = 2\times\left(\frac{D}{2}\right)\sin\left(\frac{\theta}{2}\right) = D\sin\left(\frac{\theta}{2}\right)$$

We know that $\theta = \dfrac{360°}{T}$

$$\therefore \quad p = D\sin\left(\frac{360°}{2T}\right) = D\sin\left(\frac{180°}{T}\right)$$

or

$$D = p\,\mathrm{cosec}\left(\frac{180°}{T}\right)$$

The sprocket outside diameter (D_o), for satisfactory operation is given by

$$D_o = D + 0.8\,d_1$$

where d_1 = Diameter of the chain roller.

Note: The angle θ/2 through which the link swings as it enters contact is called **angle of articulation.**

21.5 Velocity Ratio of Chain Drives

The velocity ratio of a chain drive is given by

$$V.R. = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

where N_1 = Speed of rotation of smaller sprocket in r.p.m.,

N_2 = Speed of rotation of larger sprocket in r.p.m.,

T_1 = Number of teeth on the smaller sprocket, and

T_2 = Number of teeth on the larger sprocket.

The average velocity of the chain is given by

$$v = \frac{\pi D N}{60} = \frac{T\, p\, N}{60}$$

where D = Pitch circle diameter of the sprocket in metres, and

p = Pitch of the chain in metres.

21.6 Length of Chain and Centre Distance

An open chain drive system connecting the two sprockets is shown in Fig. 21.3.

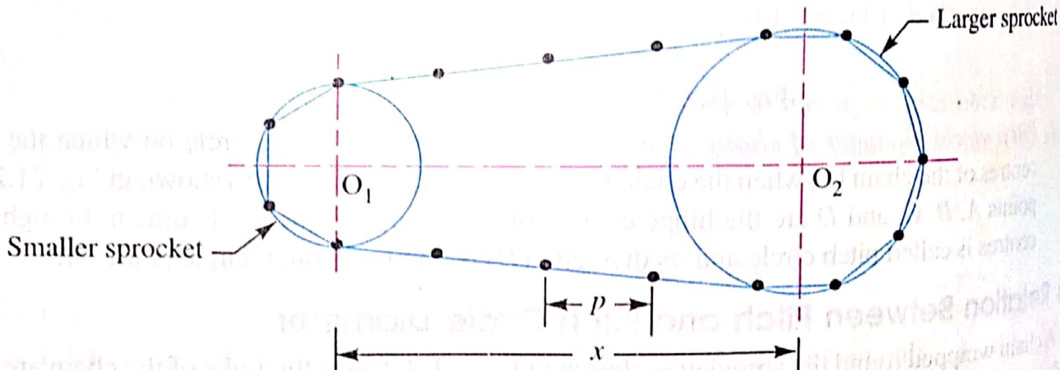


Fig. 21.3. Length of chain.

Let T_1 = Number of teeth on the smaller sprocket,

T_2 = Number of teeth on the larger sprocket,

p = Pitch of the chain, and

x = Centre distance.

The length of the chain (L) must be equal to the product of the number of chain links (K) and the pitch of the chain (p). Mathematically,

$$L = K.p$$

The number of chain links may be obtained from the following expression, *i.e.*

Pg no 7.75 PSG DDB

$$K = \frac{T_1 + T_2}{2} + \frac{2x}{p} + \left[\frac{T_2 - T_1}{2\pi}\right]^2 \frac{p}{x}$$

The value of K as obtained from the above expression must be approximated to the nearest even number.

The centre distance is given by

$$x = \frac{p}{4}\left[K - \frac{T_1 + T_2}{2} + \sqrt{\left(K - \frac{T_1 + T_2}{2}\right)^2 - 8\left(\frac{T_2 - T_1}{2\pi}\right)^2}\right]$$

In order to accommodate initial sag in the chain, the value of the centre distance obtained from the above equation should be decreased by 2 to 5 mm.

Notes: 1. The minimum centre distance for the velocity transmission ratio of 3, may be taken as

$$x_{min} = \frac{d_1 + d_2}{2} + 30 \text{ to } 50 \text{ mm}$$

where d_1 and d_2 are the diameters of the pitch circles of the smaller and larger sprockets.

2. For best results, the minimum centre distance should be 30 to 50 times the pitch.

3. The minimum centre distance is selected depending upon the velocity ratio so that the arc of contact of the chain on the smaller sprocket is not less than 120°. It may be noted that larger angle of arc of contact ensures a more uniform distribution of load on the sprocket teeth and better conditions of engagement.

21.7 Classification of Chains

The chains, on the basis of their use, are classified into the following three groups:

1. Hoisting and hauling (or crane) chains,
2. Conveyor (or tractive) chains, and
3. Power transmitting (or driving) chains.

These chains are discussed, in detail, in the following pages.

21.8 Hoisting and Hauling Chains

These chains are used for hoisting and hauling purposes and operate at a maximum velocity of 0.25 m / s. The hoisting and hauling chains are of the following two types:

1. ***Chain with oval links.*** The links of this type of chain are of oval shape, as shown in Fig. 21.4 (*a*). The joint of each link is welded. The sprockets which are used for this type of chain have receptacles to receive the links. Such type of chains are used only at low speeds such as in chain hoists and in anchors for marine works.

(*a*) Chain with oval links. (*b*) Chain with square links.

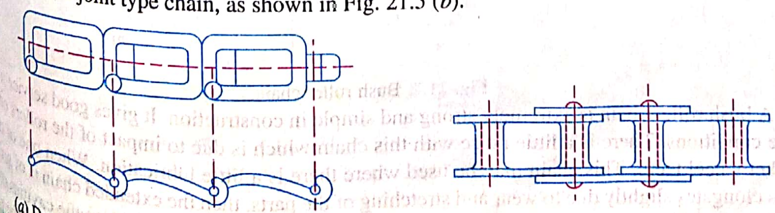
Fig. 21.4. Hoisting and hauling chains.

2. ***Chain with square links.*** The links of this type of chain are of square shape, as shown in Fig. 21.4 (*b*). Such type of chains are used in hoists, cranes, dredges. The manufacturing cost of this type of chain is less than that of chain with oval links, but in these chains, the kinking occurs easily on overloading.

21.9 Conveyor Chains

These chains are used for elevating and conveying the materials continuously at a speed upto 2 m / s. The conveyor chains are of the following two types:

1. Detachable or hook joint type chain, as shown in Fig. 21.5 (*a*), and
2. Closed joint type chain, as shown in Fig. 21.5 (*b*).



(*a*) Detachable or hook joint type chain. (*b*) Closed joint type chain.

Fig. 21.5. Conveyor chains.

The conveyor chains are usually made of malleable cast iron. These chains do not have smooth running qualities. The conveyor chains run at slow speeds of about 0.8 to 3 m / s.

21.10 Power Transmitting Chains

These chains are used for transmission of power, when the distance between the centres of shafts is short. These chains have provision for efficient lubrication. The power transmitting chains are of the following three types.

1. ***Block or bush chain.*** A block or bush chain is shown in Fig. 21.6. This type of chain was used in the early stages of development in the power transmission.

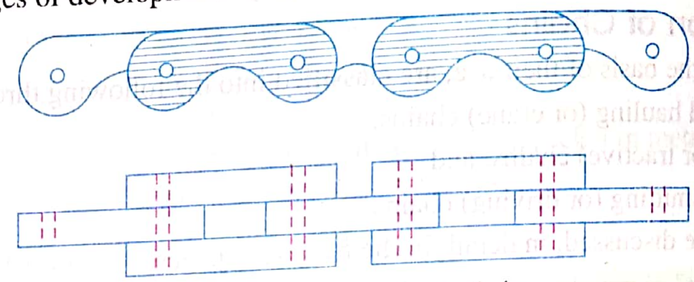


Fig. 21.6. Block or bush chain.

It produces noise when approaching or leaving the teeth of the sprocket because of rubbing between the teeth and the links. Such type of chains are used to some extent as conveyor chain at small speed.

2. ***Bush roller chain.*** A bush roller chain as shown in Fig. 21.7, consists of outer plates or pin link plates, inner plates or roller link plates, pins, bushes and rollers. A pin passes through the bush which is secured in the holes of the roller between the two sides of the chain. The rollers are free to rotate on the bush which protect the sprocket wheel teeth against wear. The pins, bushes and rollers are made of alloy steel.

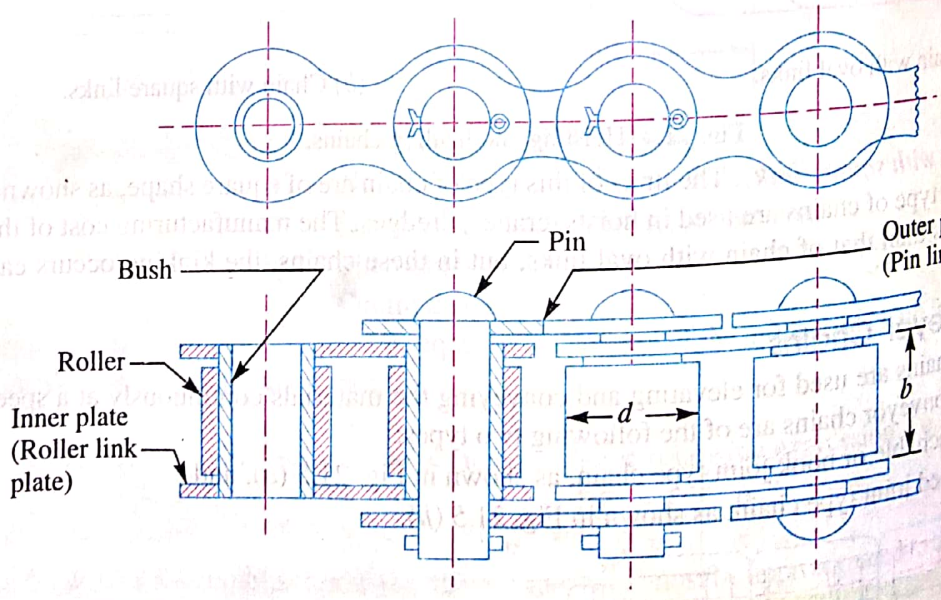
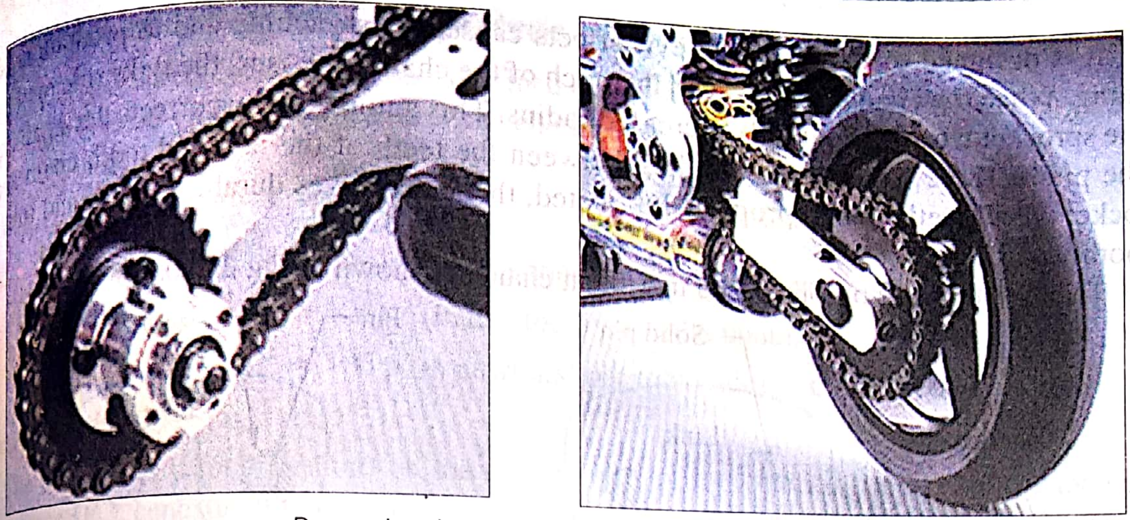


Fig. 21.7. Bush roller chain.

A bush roller chain is extremely strong and simple in construction. It gives good service under severe conditions. There is a little noise with this chain which is due to impact of the rollers on the sprocket wheel teeth. This chain may be used where there is a little lubrication. When one of these chains elongates slightly due to wear and stretching of the parts, then the extended chain is of greater pitch than the pitch of the sprocket wheel teeth. The rollers then fit unequally into the cavities of the wheel. The result is that the total load falls on one teeth or on a few teeth. The stretching of the parts increase wear of the surfaces of the roller and of the sprocket wheel teeth.



Rear wheel chain drive of a motorcycle

The roller chains are standardised and manufactured on the basis of pitch. These chains are available in single-row or multi-row roller chains such as simple, duplex or triplex strands, as shown in Fig. 21.8.

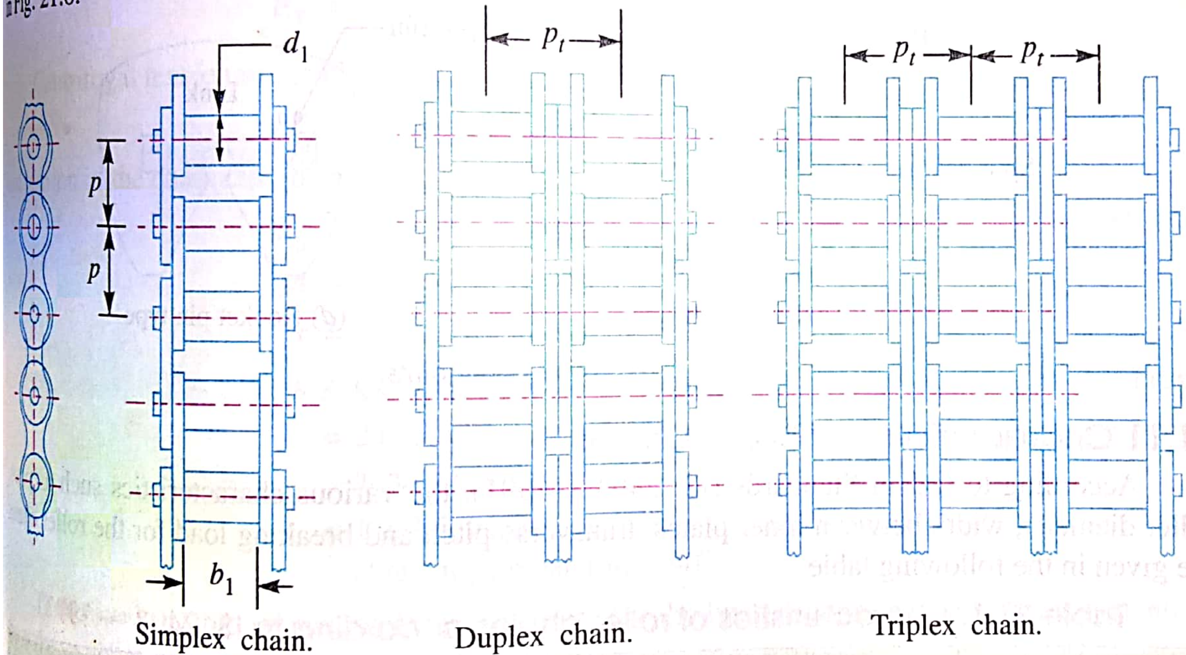


Fig. 21.8. Types of roller chain.

3. ***Silent chain.*** A silent chain (also known as ***inverted tooth chain***) is shown in Fig. 21.9.

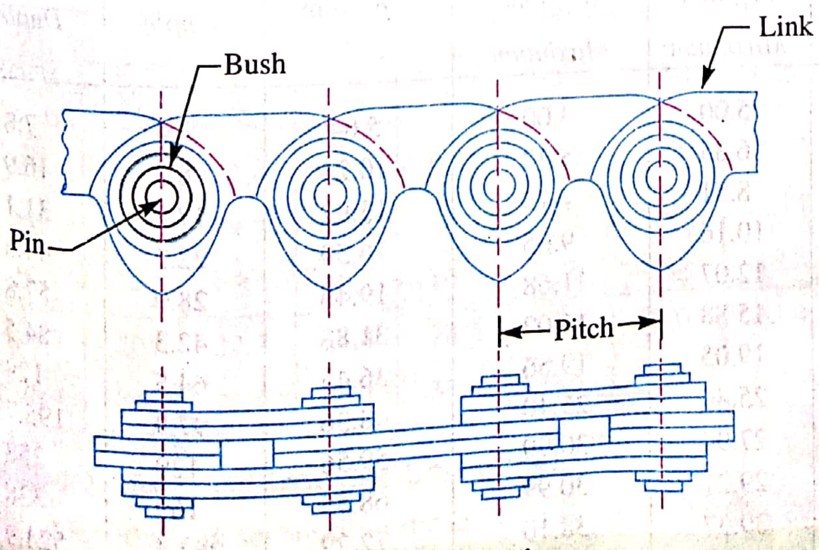
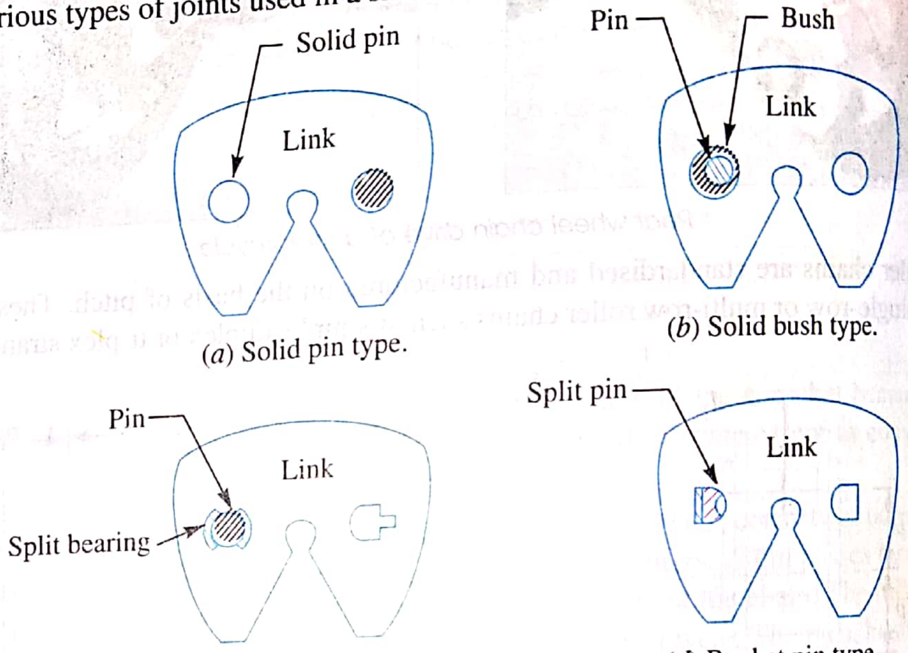


Fig. 21.9. Silent chain.

It is designed to eliminate the evil effects caused by stretching and to produce noiseless running. When the chain stretches and the pitch of the chain increases, the links ride on the teeth of the sprocket wheel at a slightly increased radius. This automatically corrects the small change in the pitch. There is no relative sliding between the teeth of the inverted tooth chain and the sprocket wheel teeth. When properly lubricated, this chain gives durable service and runs very smoothly and quietly.

The various types of joints used in a silent chain are shown in Fig 21.10.



(*a*) Solid pin type.

(*b*) Solid bush type.

(*c*) Split bush type.

(*d*) Rocket pin type.

Fig. 21.10. Silent chain joints.

21.11 Characteristics of Roller Chains

According to Indian Standards (IS: 2403 —1991), the various characteristics such as pitch, roller diameter, width between inner plates, transverse pitch and breaking load for the roller chains are given in the following table.

Table 21.1. Characteristics of roller chains according to IS: 2403 — 1991.

ISO Chain number	*Pitch (p) mm*	*Roller diameter (d_1) mm Maximum*	*Width between inner plates (b_1) mm Maximum*	*Transverse pitch (p_1) mm*	*Breaking load (kN) Minimum*		
					Simple	*Duplex*	*Triplex*
05 B	8.00	5.00	3.00	5.64	4.4	7.8	11.1
06 B	9.525	6.35	5.72	10.24	8.9	16.9	24.9
08 B	12.70	8.51	7.75	13.92	17.8	31.1	44.5
10 B	15.875	10.16	9.65	16.59	22.2	44.5	66.7
12 B	19.05	12.07	11.68	19.46	28.9	57.8	86.7
16 B	25.4	15.88	17.02	31.88	42.3	84.5	126.8
20 B	31.75	19.05	19.56	36.45	64.5	129	193.5
24 B	38.10	25.40	25.40	48.36	97.9	195.7	293.6
28 B	44.45	27.94	30.99	59.56	129	258	387
32 B	50.80	29.21	30.99	68.55	169	338	507.10
40 B	63.50	39.37	38.10	72.29	262.4	524.9	787.3
48 B	76.20	48.26	45.72	91.21	400.3	800.7	1201

Transmission ratio $i = \frac{Z_2}{Z_1} = \frac{n_1}{n_2}$

Z_1, number of teeth on sprocket pinion

Z_2, number of teeth on sprocket wheel

n_1, speed of rotation of pinion, rpm

n_2, speed of rotation of wheel, rpm

Preferred transmission ratio, i

1 1.12 1.25 1.4 1.6 1.8 2 2.25 3.15 4 4.5 5 5.6 6.3 7.1

Recommended Z_1

Transmission ratio, i	1—2	2—3	3—4	4—5	5—7
Number of teeth on sprocket, Z_1	30—27	27—25	25—23	23—21	21—17

Where space is a problem, Z_1 minimum = 7

Number of teeth on sprocket wheel, Z_2

$Z_2 = i \cdot Z_1$

$Z_{2\,max} = 100$ to 120

Note: When Z_2 is very large, chain slips off the sprocket for a small pull.

Maximum speed of rotation of the pinion, $n_{1\,max}$ rpm

Number of teeth on sprocket pinion Z_1	p, pitch, mm		
	9.525	12.7	15.875
7	2800	2600	2000
9	2800	2400	1800
15	2400	2400	1800
21	2400	2100	1500
27	2100	1800	1300
35	1800	1600	1200
45	1600	1400	1000

Centre distance, a, mm

Optimum centre distance $a = (30 \text{ to } 50)$ p

a, centre distance, mm

p, pitch of chain, mm

Minimum centre distance, a_{min}

Transmission ratio, i	Minimum centre distance, a_{min}, mm
3	a' +(30 to 50), mm
3—4	1.2 a'
4—5	1.3 a'
5—6	1.4 a'
6—7	1.5 a'

where

$$a' = \frac{d_{a1} + d_{a2}}{2}$$

d_{a1}, tip diameter of sprocket pinion, mm

d_{a2}, tip diameter of sprocket wheel, mm

tip diameter $d_a = \dfrac{p}{\tan \dfrac{180°}{Z}} + 0.6p$

Maximum centre distance, a_{max}, mm

$a_{max} = 80\ p$ p, pitch of chain, mm

Relationship between centre distance and length of chain

Nomenclature	Symbol	Formulae
Length of continuous chain in multiples of pitches (i. e. approximate number of links)	l_p	$l_p = 2a_p + \dfrac{Z_1 + Z_2}{2} + \dfrac{\left(\dfrac{Z_2 - Z_1}{2\pi}\right)^2}{a_p}$ to be corrected to an even number
Approximate centre distance in multiples of pitches	a_p	$a_p = \dfrac{a_o}{p}$ a_o, initially assumed centre distance, mm p, pitch, mm
Final centre distance corrected to even number of pitches	a	$a = \dfrac{e + \sqrt{e^2 - 8m}}{4}\ p$ $e = l_p - \dfrac{Z_1 + Z_2}{2}$ $m = \left(\dfrac{Z_2 - Z_1}{2\pi}\right)^2$, constant For value of m, see page 7.76
Length of chain	l	$l = l_p \cdot p$
Centre distance (decrement) allowance to accommodate initial chain sag	Δ	$\Delta = \frac{1}{2}\left[f - \left(\dfrac{Z_2 - Z_1}{2\pi a_p}\right)^2 \cdot p\right] \approx 0.5\ f$ f, chain sag of driven side, mm $f = 0.02\ a$, mm

21.20 SOLVED PROBLEMS :

AS PER MANUFACTURER'S DATA :

Problem 21.1:

Select a flat belt to drive a mill at 250 rpm from a 10 kW, 730 rpm motor. Centre distance is to be around 2m. The mill shaft pulley is of 1 m diameter. **(Anna University)**

Solution :

Design power = Rated power × Service factor × Arc. of contact factor = $P \times K_s \times K_a$.

Rated power $P = 10$ kW

Service factor $K_s = 1.3$ (Assuming heavy duty intermittent load).

(Table 21.5) (PSG 7.53) (JDB 21.5)

Arc of contact $\theta = 180^\circ - \left(\frac{D-d}{C}\right)60^\circ$

$$D = 1\text{ m} = 1000\text{ mm}$$

$$i = \frac{n}{N} = \frac{730}{250} = 2.92$$

$$d = \frac{1000}{2.92} = 342.46\text{ mm} = 345\text{ mm (Say)}$$

Arc of contact $\theta = 180^\circ - \left[\frac{1000-345}{2000}\right]60^\circ = 160.4^\circ$

Arc of contact factor $K_a = 1.08$ (Table 3.6) (PSG 7.54, JDB 21.5)

Now

Design power, $P_d = 10 \times 1.3 \times 1.08 = 14.04\text{ kW}$

Select Dunlop Fort 949 g fabric belting. For finding the number of plies, let us find the belt speed.

Belt speed $v = \frac{\pi d n}{60 \times 1000} = \frac{\pi \times 345 \times 730}{60 \times 1000} = 13.2\text{ m/s.}$

Select 6 ply belt. (Table 3,7) (PSG 7,52, JDB 21.5)

Belt Rating :

Belt capacity, $B_r = 0.0289$ kW per m.m. width per ply at 180° arc of contact and at 10 m/s.

$$= 0.0289 \times \frac{13.2}{10} \times \frac{160.4}{180} \times 6\text{ kW per m.m. width}$$

$$= 0.204\text{ kW per m.m width}$$

Total width of belt $= \frac{\text{Design power}}{\text{Belt Rating}} = \frac{P_d}{B_r} = \frac{14.04}{0.204} = 69.0$ mm

Next higher standard belt width $= 112$ mm (Table. 21.S) (PSG 7.52, JDB 21.6)

Length of belt, $L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$

$$= (2 \times 2) + \frac{\pi}{2}(1 + 0.345) + \frac{(1-0.345)^2}{4 \times 2}$$

$$= 4 + \frac{\pi}{2}(1.345) + \frac{(0.655)^2}{8} = 6.166\text{ m} = 6166\text{ mm}$$

Initial tension to be provided to the belt for firm grip

$$= 1\% \text{ of } L = 0.01 \times 6166 = 62 \text{ mm}$$

Length after standard deduction for initial tension (i.e.; after reducing 1% of L)

$$= 6166 - 62 = 6104 \text{ mm} = 6100 \text{ mm}$$

Pulley width $= 112 + 13 = 125$ mm (standard value of itself is 125 mm)

(Table 21.9, 21.10) (PSG 7.54, JDB 21.7)

Specifications :

Dunlop fort 949 g fabric belting of 112 mm width may be selected.

Pulley width = 125 mm

Length of belt = 6100 mm

Problem 21.2 :

Design a belt drive to transmit 30 H.P. at 740 rpm to an aluminium rolling machine, the speed ratio being 3.0; The distance between the pulleys is 3 metre. Diameter of the rolling machine pulley is 1.2 metre.

(Madras University)

Galling is a form of wear caused by adhesion between sliding surfaces. When a material galls, some of it is pulled with the contacting surface, especially if there is a large amount of force compressing the surfaces together. Galling is caused by a combination of friction and adhesion between the surfaces, followed by slipping and tearing of crystal structure beneath the surface. This will generally leave some material stuck or even friction welded to the adjacent surface, whereas the galled material may appear gouged with balled-up or torn lumps of material stuck to its surface.

Galling is most commonly found in metal surfaces that are in sliding contact with each other. It is especially common where there is inadequate lubrication between the surfaces. However, certain metals will generally be more prone to galling, due to the atomic structure of their crystals. For example, aluminium is a metal that will gall very easily, whereas annealed (softened) steel is slightly more resistant to galling. Steel that is fully hardened is very resistant to galling.

Galling is a common problem in most applications where metals slide while in contact with other metals. This can happen regardless of whether the metals are the same or of different kinds. Alloys such as brass and bronze are often chosen for bearings, bushings, and other sliding applications because of their resistance to galling, as well as other forms of mechanical abrasion.

Galling is adhesive wear that is caused by microscopic transfer of material between metallic surfaces, during transverse motion (sliding). It occurs frequently whenever metal surfaces are in contact, sliding against each other, especially with poor lubrication. It often occurs in high load, low speed applications, but also in high-speed applications with very little load. Galling is a common problem in sheet metal forming, bearings and pistons in engines, hydraulic cylinders, air motors, and many other industrial operations. Galling is distinct from gouging or scratching in that it involves the visible transfer of material as it is adhesively pulled (mechanically spalled) from one surface, leaving it stuck to the other in the form of a raised lump (gall). Unlike other forms of wear, galling is usually not a gradual process, but occurs quickly and spreads rapidly as the raised lumps induce more galling. It can often occur in screws and bolts, causing the threads to seize and tear free from either the fastener or the hole. In extreme cases, the bolt may seize without stripping the threads, which can lead to breakage of the fastener or the tool turning it. Threaded inserts of hardened steel are often used in metals like aluminium or stainless steel that can gall easily.[1]

Galling requires two properties common to most metals, cohesion through metallic-bonding attractions and plasticity (the ability to deform without breaking). The tendency of a material to gall is affected by the ductility of the material. Typically, hardened materials are more resistant to galling whereas softer materials of the same type will gall more readily. The propensity of a material to gall is also affected by the specific arrangement of the atoms, because crystals arranged in a face-centered cubic (FCC) lattice will usually allow material-transfer to a greater degree than a body-centered cubic (BCC). This is because a face-centered cubic has a greater tendency to produce dislocations in the crystal lattice, which are defects that allow the lattice to shift, or "cross-slip," making the metal more prone to galling. However, if the metal has a high number of stacking faults (a difference in stacking sequence between atomic planes) it will be less apt to cross-slip at the dislocations. Therefore, a material's resistance to galling is usually determined by its stacking-fault energy. A material with high stacking-fault energy, such as aluminium or titanium, will be far more susceptible to galling than materials with low stacking-fault energy, like copper, bronze, or gold. Conversely, materials with a hexagonal close packed (HCP) structure and a high *c/a* ratio, such as cobalt-based alloys, are extremely resistant to galling.[2]

Galling occurs initially with material transfer from individual grains, on a microscopic scale, which become stuck or even diffusion welded to the adjacent surface. This transfer can be enhanced if one or both metals form a thin layer of hard oxides with high coefficients of friction, such as those found on aluminum or stainless-steel. As the lump grows it pushes against the adjacent material and begins forcing them apart, concentrating a majority of the friction heat-energy into a very small area. This in turn causes more adhesion and material build-up. The localized heat increases the plasticity of the galled surface, deforming the metal, until the lump breaks through the surface and begins plowing up large amounts of material from the galled surface. Methods of preventing galling include the use of lubricants like grease and oil, low-friction coatings and thin-film deposits like molybdenum disulfide or titanium nitride, and increasing the surface hardness of the metals using processes such as case hardening and induction hardening.

Selection of a chain drive

Design a chain drive to actuate a compressor from a 10KW electric motor running at 960 rpm. The compressor speed is to be 350 rpm. Minimum centre distance should be 0.5m. Motor is mounted on an auxiliary bed. Compressor is to work for 8 hours/day.

Given Data:

Power = 10 kW
Motor Speed = 960 rpm
Compressor speed = 350 rpm
Centre distance, a = 0.5 m
Motor mounted on auxiliary bed
Service required: 8 hrs/day.

Solution:

Let the operating chain may be a roller chain (PSG 7.71)
Since the optimum centre distance, 'a' is 30p to 50p (PSG 7.74) assume a=35p where 'p' is the pitch of the chain.
P = (a/35) = (500/35) = 14.3 mm
The next higher standard pitch, p = 15.875 mm is selected from PSG 7.72Solution:

Transmission ratio, i = (960/350)= 2.74
For i=2.74, the number of teeth on pinion sprocket, Z1= 25 (assumed) (PSG 7.74)
The number of teeth on wheel sprocket Z2=2.74 x 25 = 68.5 = 69
The power that can be transmitted on the basis of breaking load is given by
N = (Q v) / (102 n k

- Where N = 10 kW
- Q = breaking load, N
- v = chain velocity in m/s
- n = allowable factor of safety (minimum
- and k_s =Service factor

- $k_s = k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot k_5 \cdot k_6$
- k_1 = 1.5 (load heavy shock)
- k_2 = 1.0 (adjustable supports)
- k_3 = 1.0 (a_p=30 to 50 p)
- k_4 = 1.0 (the line joining centres is inclined to 60◦ to the horizontal)
- k_5 = 1.0 (drop lubrication assumed)
- k_6 = 1.0 (8 hours/day service)
- k_s = 1.5 x 1.0 x 1.0 x 1.0 x 1.0 x 1.0 = 1.5
- The factor of safety, n = 11 for k_s =1 and Z_1=25 (PSG 7.77)
- Corrected factor of safety, n = 11 x 1.5 = 16.5 for k_s =1.5 and Z_1=25

- Chain velocity, v = (Z_1 n_1 p) / (60x1000)
- V = (25 x 960 x 15.875) / (60 x 1000)
- = 6.35 m/s
- The breaking load, Q = (102 n k_s N) / v
- = (102x16.5x1.5x10) / 6.35
- = 3975.5 kgf (or 39755 N)
- We need to select a chain having a breaking load higher than this value from PSG 7.72
- For the pitch value of 15.875 mm, DR50 Rolon Chain is selected with a breaking load of 4440 kgf (or 44400 N).

<u>Checking for actual factor of safety</u>

Actual factor of safety = $\frac{Q}{\Sigma P}$ where $\Sigma P = P_t + P_c + P_s$

$P_t = \frac{102\,N}{v}$; N in kW $P_t = \frac{102 \times 16.5}{6.35}$ = [illegible] kgf

$P_c = \frac{w v^2}{g}$; w in kgf/m = 1.78 kgf/m (PSG 7.74)

$P_c = \frac{1.78 \times 6.35^2}{9.81} = 7.31$ kgf

P_s = Tension due to sagging of chain = k.w.a

where k = sagging coefficient = 6

(PSG 7.78 ; position of chain drive inclined to 40°)

$P_3 = 4 \times 1.78 \times 0.555$ $\quad$ $(a = 35p = 35 \times 15.875$
$\quad = 3.95 \text{ kgf}$ $\qquad\qquad$ $= 555 \text{ mm} = 0.555 \text{ m})$

$\therefore \Sigma F = P_1 + P_2 + P_3 = 160.65 + 7.31 + 3.95 = 171.84 \text{ kgf}$

Actual factor of safety $= \dfrac{Q}{\Sigma F} = \dfrac{4445}{171.84} = 25.8$

Since the actual factor of safety is more than the adopted minimum value, the chain selection is correct.

Checking for induced bearing stress

The induced bearing stress over the projected area of the chain is given by

$$\sigma = \frac{102\, k_2\, N}{A\, v} = \frac{102 \times 1 \times 10}{1.6 \times 3.58} \qquad (A = 1.6 \text{ cm}^2 \text{ or } 160 \text{ mm}^2 \text{ PSG } 7.74)$$

$$= 178 \text{ kgf/cm}^2 = 1.78 \text{ kgf/mm}^2$$

Allowable bearing stress $= 2.24 \text{ kgf/mm}^2$ for $k_2 = 1$ and $Z_1 = 15$ to [illegible]

$= 2.56 \text{ kgf/mm}^2$ for $k_2 = 1.5$ and $Z_1 = 15$ to [illegible]

Hence the chain selection is correct

The length of chain in terms of pitches is obtained as

$$l_p = 2a_p + \frac{(z_1+z_2)}{2} + \frac{\left\{\frac{z_2-z_1}{2\pi}\right\}^2}{a_p} \quad \text{(PSG 7.74)}$$

$$= 2(\text{[illegible]}) + \frac{(28+64)}{2} + \frac{\left\{\frac{64-28}{2\pi}\right\}^2}{\text{[illegible]}}$$

$= 118.6$ pitches ≈ 118 pitches

Actual length of chain $= 118 \times 15.875 = 1873$ mm

The corrected centre distance

$$a = \left(\frac{E + \sqrt{E^2 - 8m}}{4}\right) \times p \quad \text{(PSG 7.75)}$$

Where, $E = l_p - \left(\frac{z_1+z_2}{2}\right) = 118 - \left(\frac{28+64}{2}\right) = 72$

$m = 46$ (for $z_2 - z_1 = 64 - 28 = 36$
PSG 7.75)

$$\therefore a = \left(\frac{72 + \sqrt{72^2 - 8(46)}}{4}\right) \times 15.875 = 561 \text{ mm}$$

Centre distance allowance to accommodate initial chain sag, $\Delta a = 0.01\,a$ (PSG 7.75)

$\Delta a = 0.5 \times 0.01 \times a = 0.5 \times 0.01 \times 1120$

$= 5.6$ mm

Hence, required distance $= 561 - 5.6 = 555.4$ mm

$\simeq 555$ mm

Pitch diameters of pinion & wheel sprockets

Pitch diameter of pinion sprocket, $d_1 = \dfrac{p}{\sin\left(\frac{180}{z_1}\right)}$

$$d_1 = \frac{15.875}{\sin\left(\frac{180}{25}\right)} = 126.7 \text{ mm}$$

Pitch diameter of wheel sprocket, $d_2 = \dfrac{p}{\sin\left(\frac{180}{z_2}\right)}$

$$d_2 = \frac{p}{\sin\left(\frac{180}{z_2}\right)} = \frac{15.875}{\sin\left(\frac{180}{64}\right)} = 324.6 \text{ mm}$$

Specifications

Type of chain : 10A-2 DR50 Roller chain

Centre distance = 555 mm

Number of teeth of pinion sprocket = 25

" " " wheel " = 64

Length of chain = 1880 mm

Pitch dia. of pinion sprocket = 127 mm

" " wheel " = 324 mm

———— x ————

Problem 4.2: *A pair of helical gears with 30° helix angle is used to transmit 15 kW at 10,000 rpm. The velocity ratio is 4:1; Both the gears are made of case hardened steel with a static strength of 100 N/mm². The gears are 20° stub and the pinion is to have 24 teeth. The face width is 14 times the module. Find the module and face width for strength.* **(FAQ)**

Given Data:

Helix angle $= \beta = 30°$

$P = 15 \text{ kW} = 15 \times 10^3 \text{ W} = 20.38 \text{ hp}$ (1 hp = 736 W)

$i = 4:1$

$N = 10{,}000$ rpm

Both the gears are made of case hardened steel.

Static strength $= [\sigma_b] = 100 \text{ N/mm}^2 \times 10$

$= 1000 \text{ kgf/cm}^2$

[Convert into kgf/cm²

i.e., $\text{N/mm}^2 \times 10 = 1 \text{ kgf/cm}^2$]

Pressure angle $= 20°$ involute stub.

No. of teeth on pinion $= 24$

Face width $= b = 14$ times module

$$b = 14 \times m_n$$

Step 1

$$F_s = \text{Beam strength} = [\sigma_b]\, b\, y_v\, \pi\, m_n$$

We know that,

$$y_v = 0.175 - \frac{0.95}{Z_v}$$

$$Z_v = \frac{z}{\cos^3 \beta} = \frac{24}{\cos^3 30°} = 36.95$$

$$y_v = 0.1492$$

$$F_s = 1000 \times 14\,(m_n) \times 0.1492 \times \pi \times m_n$$

$$= (6562.15)\, m_n^2 - \text{kgf} \quad \text{... (1)}$$

Transmitted load $= F_t = \text{HP} \times \dfrac{75}{v_m}$

$$v_m = \frac{\pi Z_1 m_n N_1}{100 \times 60} \text{ m/s}$$

(m_n is in cm)

$$= \frac{\pi \times 24 \times m_n \times 10{,}000}{100 \times 60}$$

$$v_m = 125.66\,(m_n) \text{ m/s}$$

$$V_m = 7539.82\,(m_n) \text{ m/min}$$

$$F_t = 20.38 \times \frac{75}{125.66\,(m_n)}$$

$$= \frac{(12.16)}{m_n}$$

Dynamic load $= F_d = F_t \times C_v$

where $C_v = \dfrac{5.5 + \sqrt{v_m}}{5.5} =$ For precision wheels, $V_m > 20$ m

$$= \frac{5.5 + \sqrt{125.66\, m_n}}{5.5}$$

$$\therefore F_d = \left(\frac{12.16}{m_n}\right) \frac{5.5 + \sqrt{125.66\, m_n}}{5.5}$$

$$= \frac{2.2115}{m_n}\left(5.5 + \sqrt{125.66\, m_n}\right) \quad \ldots(2)$$

To find normal module m_n, equate the above equations (1) and (2)

$$6562.15\, m_n^2 = \frac{2.2115}{m_n}\left(5.5 + \sqrt{125.66\, m_n}\right)$$

$$2967.28\, m_n^3 = 5.5 + \sqrt{125.66\, m_n}$$

$$2967.28\, m_n^3 = 5.5 + \sqrt{125.66\, m_n}$$

Calculate m_n, by using trail and error method.

Trail 1

Assume $m_n = 0.5$ cm

LHS = 371.1175; RHS = 13.42

LHS > RHS

$\therefore$ Not satisfactory.

Trail 2

$m_n = 0.3$ cm

LHS = 80.162; RHS = 11.639

LHS > RHS

$\therefore$ Not satisfactory.

Trail 3

$m_n = 0.2$

LHS = 23.75 and RHS = 10.513

LHS > RHS

$\therefore$ Not satisfactory

Trail 4

$m_n = 0.15$ cm

LHS = 10.01; RHS = 9.84

LHS ≈ RHS

Therefore, m_n = Normal module

= 0.15 cm

$\boldsymbol{m_n = 1.5}$ **mm**

$$m_t = \frac{m_n}{\cos \beta} = \frac{1.5}{\cos 30°} = 1.732 \text{ mm}$$

Base width $= b = 14\, m_n = 14 \times 1.5$

$\boldsymbol{b = 21}$ **mm**

The pitch diameter of pinion sprocket, (d)

$$d = \frac{p}{\sin\left(\frac{180°}{Z_1}\right)} = \frac{15.875}{\sin\left(\frac{180°}{25}\right)}$$

$\therefore$ $d = 127$ mm

and

The pitch diameter of wheel sprocket, (D)

$$D = \frac{p}{\sin\left(\frac{180°}{Z_2}\right)} = \frac{15.875}{\sin\left(\frac{180°}{72}\right)}$$

$\therefore$ $D = 364$ mm

Specifications

Type of chain = 10 A – 2 DR 50 Rolon chain

Centre distance = 505 mm

number of teeth of sprocket pinion = 25

number of teeth of sprocket wheel = 72

Length of chain = 1810 mm

Pitch diameter of pinion sprocket = 127 mm

Pitch diameter of wheel sprocket = 364 mm

Problem 5.35: *Design a chain drive to actuate a compressor from a 10 kW electric motor at 960 rpm. The Compressor speed is to be 350 rpm. Minimum center distance should be 0.5 m. Compressor is to work for 8 hours/day*

Given data

Power of motor $(N) = 10$ kW

Speed of motor $(N_1) = 960$ rpm

Speed of the compressor $(N_2) = 350$ rpm

Minimum centre distance $(a) = 500$ mm

Solution

Step 1: Calculation of transmission ratio

From **PSG Data book, Pg.No: 7.74**

Transmission ratio $(i) = \frac{Z_2}{Z_1} = \frac{N_1}{N_2}$

$$\therefore (i) = \frac{N_1}{N_2} = \frac{960}{350} = 2.74$$

Step 2: Number of Teeth

From **PSG Data book, Pg.No. 7.74** (or) **KM Databook Pg.No. 339**

For $i = 2$ to 3; $Z_1 = 25$ to 27

$i = 3$ to 4; $Z_1 = 23$ to 25

Take $Z_1 = 25$ to 27 (select any odd number of teeth)

Select $Z_1 = 27$ teeth (number of teeth on sprocket pinion)

$Z_2 = i\,Z_1 = 2.74 \times 27$

$\Rightarrow Z_2 = 73.98 \simeq 74$

$\therefore Z_2 = 74$ teeth

Step 3: Selection of pitch

From **PSG Design data book, Pg.No: 7.74**

Optimum centre distance $(a) = (30 \text{ to } 50)\, p$

$$(\text{Pitch})_{max} = \frac{500}{30} = 16.66$$

$$(\text{Pitch})_{min} = \frac{500}{50} = 10$$

Select standard pitch from PSG Data book **Pg.No. 7.74**

Take any standard pitch between 10 mm and 16.66 mm

$\therefore$ select pitch $(p) = 15.875$ mm

Step 4: Selection of chain number

From **PSG Data book, Pg.No. 7.72**

The available chain number are 10 A and 10 B

Select $10\,A - 2$ Duplex chain

From **PSG Data book, Pg.No. 7.72**

corresponding to $10\,A - 2$ and Pitch = 15.875 mm, we get

A = Bearing area = $1.4\ \text{cm}^2$

W = Weight per metre length = 1.78 kgf

Q = Breaking load = 4440 kgf

Step 5: Selection of Breaking load

From **PSG Data book, Pg.No. 7.77,**

Power transmitted on the basis of breaking load

$$N = \frac{Q.v}{102n\ k_s}$$

We know that $v = \dfrac{Z_1\, p\, N_1}{60 \times 1000} = \dfrac{27 \times 15.875 \times 960}{60 \times 1000}$

$$v = 6.858\ \text{m/s}$$

Since specific conditions are not given in the problem, assume $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 1$

$\therefore k_s = 1$

From PSG Data book, **Pg.No. 7.77**, corresponding to $k_s = 1, Z_1 = 15$ to 30, pitch $(p) = 15.875$,

$N_1 = 960\ \text{rpm} < 1000\ \text{rpm}$

$\therefore n = 11$ (allowable factor of safety)

$$\therefore Q = \frac{102n\ k_s\ N}{u} = \frac{102 \times 11 \times 1 \times 10}{6.858}$$

$\therefore Q = 1636.04\ \text{kgf} <$ selected chain Breaking load (4440 kgf)

Step 6

(a) calculation of length of chain

(b) final centre distance

(a) Calculation of length of chain

From **PSG Data book, Pg.No. 7.75**, (or) **KM Databook Pg. No. 301**

Length of continuous chain in multiples of pitches

$$(l_p) = 2a_p + \frac{Z_1 + Z_2}{2} + \frac{\left(\dfrac{Z_2 - Z_1}{2\pi}\right)^2}{a_p}$$

Where, $a_p = \dfrac{a_o}{p}$

$$\therefore a_p = \frac{500}{15.875}$$

$$\Rightarrow a_p = 31.49$$

$$\therefore l_p = (2 \times 31.49) + \left[\frac{27+74}{2}\right] + \frac{\left(\dfrac{74-27}{2\pi}\right)^2}{31.49}$$

$= 62.98 + 50.5 + 1.776 = 115.25 = 116$ (approximated to 116)

$\therefore$ from **PSG Data book, Pg.No. 7.75**

Length of chain $(l) = l_p \times p$

$$= 116 \times 15.875 = 1841.5\,\text{mm}$$

Take $\boldsymbol{l = 1840}$ **mm**

(b) **Final centre distance**

From **PSG Data book, Pg.No. 7.75**

Final centre distance corrected to even number of pitches 'a'

$$a = \frac{e + \sqrt{e^2 - 8\,\text{m}}}{4} \cdot P$$

where, $e = l_p - \left(\dfrac{Z_1 + Z_2}{2}\right) = 116 - \left(\dfrac{27+74}{2}\right)$

$e = 65.5$ mm

and $m = \left(\dfrac{Z_2 - Z_1}{2\pi}\right)^2 = \left(\dfrac{74-27}{2\pi}\right)^2$

$m = 55.95$

$\therefore$ Final centre distance 'a'

$$a = \frac{65.5 + \sqrt{(65.5)^2 - (8 \times 55.95)}}{4} \times 15.875 = 505.97$$

$\therefore a \simeq$ 506 mm

Step 7: Check the actual factor of safety

From **PSG Data book, Pg.No. 7.78**

Actual factor of safety $[n] = \frac{Q}{\Sigma P}$

where, Q = breaking load of chain = 4440 kgf

and $\Sigma P = P_t + P_s + P_c$

Where, P_t = Tangential force due to power transmission

From **PSG Data book, Pg.No. 7.78**

$$P_t = \frac{102\text{ N}}{v}$$

$$\therefore P_t = \frac{102 \times 10}{6.858}$$

$$= 148.73 \text{ kgf}$$

and P_c = Centrifugal tension

From **PSG Data book, Pg.No. 7.78,**

$$P_c = \frac{W v^2}{g} = \frac{1.78 \times (6.858)^2}{9.81} = 8.533 \text{ kgf}$$

and P_s = Tension due to sagging of chain

From **PSG Data book, Pg.No. 7.78**

$$P_s = \text{k.W.a}$$

where, k = Coefficient of sag

from PSG Data book, **Pg.No. 7.78**, corresponding to position of chain drive as horizontal

k = 6

$$\therefore P_c = 6 \times 1.78 \times \frac{500}{1000} = 5.34 \text{ kgf}$$

$$\therefore \Sigma P = 148.73 + 8.533 + 5.34 = 162.60 \text{ kgf}$$

$$\therefore [n] = \frac{Q}{\Sigma P} = \frac{4440}{162.60} = 27.3 > 11$$

which is greater than allowable factor of safety

∴ The design is safe.

Step 8: Checking of Allowable bearing stress

From **PSG Data book, Pg.No. 7.77**, corresponding to pitch = 15.875 mm, speed less than 1000 rpm

$$[\sigma] = 2.24 \text{ kgf/mm}^2$$

From **PSG Data book, Pg.No. 7.77,**

Power transmitted on the basic of allowable bearing stress

$$N = \frac{\sigma A v}{102 \text{ k}_s}$$

$$\therefore \sigma = \frac{102 N \text{k}_s}{A v} = \frac{102 \times 10 \times 1}{1.4 \times 6.858}$$

$$= 106.236 \text{ kgf/cm}^2 = 1.062 \text{ kgf/mm}^2 < 2.24 \text{ kgf/mm}^2$$

$$\sigma < [\sigma]$$

$\therefore$ The design is safe

Step 9

Pitch diameter of small sprocket (d)

$$d = \frac{p}{\sin \frac{180°}{Z_1}} = \frac{15.875}{\sin \frac{180°}{27}}$$

$$\boldsymbol{d = 136.74 \text{ mm}}$$

Pitch diameter of large sprocket (D)

$$D = \frac{p}{\sin \frac{180°}{Z_2}} = \frac{15.875}{\sin \frac{180}{74}}$$

$$\boldsymbol{D = 374 \text{ mm}}$$

Problem No: 2. [Dimension of the Crank pin at the big end and the Piston Pin at the Small end].

30/04/2020

Determine the dimensions of small end and big end bearings of the connecting rod for a diesel engine with the following data.

Cylinder Bore = 100 mm

Max gas Pressure = 4 MPa = 4 N/mm^2

$\left(\frac{l}{d}\right)$ ratio for Piston Pin bearing = 2

$\left(\frac{l}{d}\right)$ ratio for Crank Pin bearing = 1.3

Allowable bearing Pressure for Piston Pin bearing = 12 MPa

Allowable bearing Pressure for Crank Pin bearing = 7.5 MPa

Given Data:

$D = 100\,mm$, $P_{max} = 4\,MPa = 4\,N/mm^2$

$(l_p/d_p) = 2$ $\quad l_c/d_c = 1.3$

$(P_B)_p = 12\,MPa = 12\,N/mm^2$ & $(P_b)_c = 7.5\,MPa = 7.5\,N/mm^2$

Step 1: To Find Max Bearing Load

Force acting on the Connecting rod (OR) Load due to gas (or) steam Pressure (P_c)

$$P_c = \left(\frac{\pi D^2}{4}\right) P_{max} = \frac{\pi (100)^2}{4}(4)$$

$$P_c = 31415.93\,N$$

Step 2:- To find Piston Pin Bearing.

Std formula

$$~~P_c = \frac{\pi D^2}{4}~~ \quad ~~(1)~~$$

$$P_c = \left[\frac{\pi D^2}{4}\right] P_{max}.$$

$$P_c = d_p l_p (P_b)_p$$

where

d_p = dia of Piston Pin (OR) Inner dia of bush on the Piston Pin in mm

l_p = Length of the Piston Pin in mm

$(P_b)_p$ = Allowable bearing Pressure for the Piston Pin bush (mm)

$$\therefore 31415.93 = d_p l_p (P_b)_p = d_p (2d_p)(12)$$

(1) If $(P_b)_p$ is not given we can take it as 10 to 12.5 MPa

(2) Similarly $\frac{l_p}{d_p} = 1.5$ to 2.0

(*) not available in PSG Design Data.

$$31415.93 = 24(d_p)^2$$

$$\therefore (d_p)^2 = \frac{31415.93}{24} = 1309$$

$d_p = 36.18$ mm $\approx$ 38 mm [Next even number]

$$l_p = 2d_p = 2 \times 38 = 76\text{ mm}$$

Step 8: Crank Pin bearing

The Construction of the big end of the Connecting rod is the crank pin bearing is a lined bearing split into two halves.

The lined bushing consist of steel backing with a thin lining of bearing material like babbit / white Aluminium.

It is also designed by bearing Considerations.

$$P_c = d_c l_c (P_b)_c \quad \text{and} \quad \frac{l_c}{d_c} = 1.25 \text{ to } 1.5$$

where d_c = Dia of Crank Pin in mm

l_c = Length of the crank pin in mm

$(P_b)_c$ = Allowable bearing Pressure for the Crank Pin bush (N/mm²)

$$\therefore P_c = d_c l_c (P_b)_c = 31415.93 = d_c (1.3 d_c) 7.5$$

$$d_c^2 = \frac{31415.93}{9.75} = 3222.15$$

$$d_c = 56.76\,mm \text{ (OR) } \underline{58\,mm}$$

$$l_c = 1.3 \times 58 = 75.4\,mm \text{ (OR) } \underline{76\,mm}$$

Problem no: 3

To find

① Size of the bolt for securing the big end cap.

② Thickness of the big end cap.

The following data is given for the cap and bolts of the big end of connecting rod:

Engine speed $(N) = 1800$ rpm

Length of connecting rod $(L) = 350$ mm

Length of stroke $(l) = 175$ mm

Mass of reciprocating parts $(m_r) = 2.5$ kg

Length of crank pin $(l_c) = 76$ mm

Dia of crank pin $(d_c) = 58$ mm

Thickness of bearing bush $= 3$ mm

Permissible tensile stress for bolts $(\sigma_t) = 60$ N/mm^2

Permissible bending stress for cap $(\sigma_b) = 80$ N/mm^2

Calculate the nominal diameter of bolts and thickness of cap for the big end.

Given data: ↑

Step 1: Inertia force

$$r = \left[\frac{l}{2}\right] = \frac{175}{2} = 87.5\text{mm} = 0.0875\text{m}$$

$r =$ crank radius in metre. & $l =$ ~~crank~~ length of

n_1 = Ratio of length of the connecting rod to the crank radius

$$\boxed{n_1 = \frac{L}{r}}$$ where L = length of connecting rod in metres

Mass of Reciprocating Parts (M_r) = [Mass of Piston assembly] + $\left(\frac{1}{3}\right)^{rd}$ [mass of connecting rod]

Angular vel of crank $(\omega) = \left[\frac{2\pi N}{60}\right]$

N = crank speed in rpm.

$$n_1 = \left[\frac{L}{r}\right] = \frac{350}{87.5} = 4$$

$$\omega = \frac{2\times 3.14\times 1800}{60} = 188.5 \text{ rad/s.}$$

⊛ $(P_i)_{max} = M_r \omega^2 r \left[1 + \frac{1}{n_1}\right]$

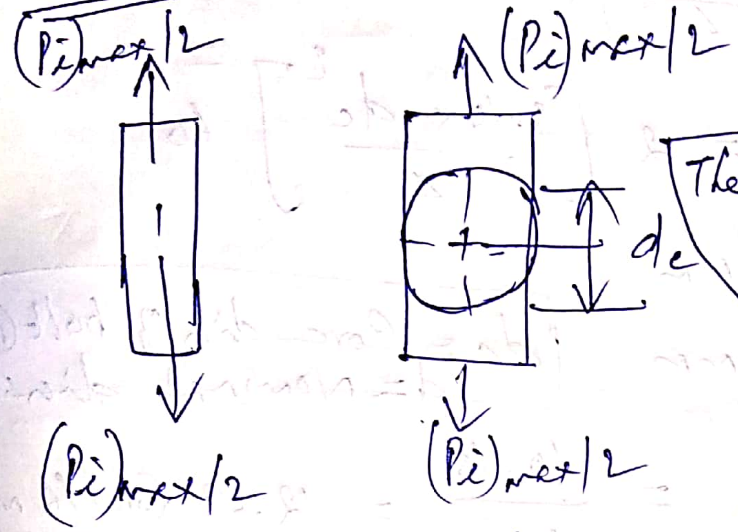
⊛ Not available in PSG Data Book

where $(P_i)_{max}$ = Inertia force on the cap or bolt (N)

$$(P_i) = 2.5 \times (188.5)^2 (0.0875) \left[1 + \frac{1}{4}\right]$$

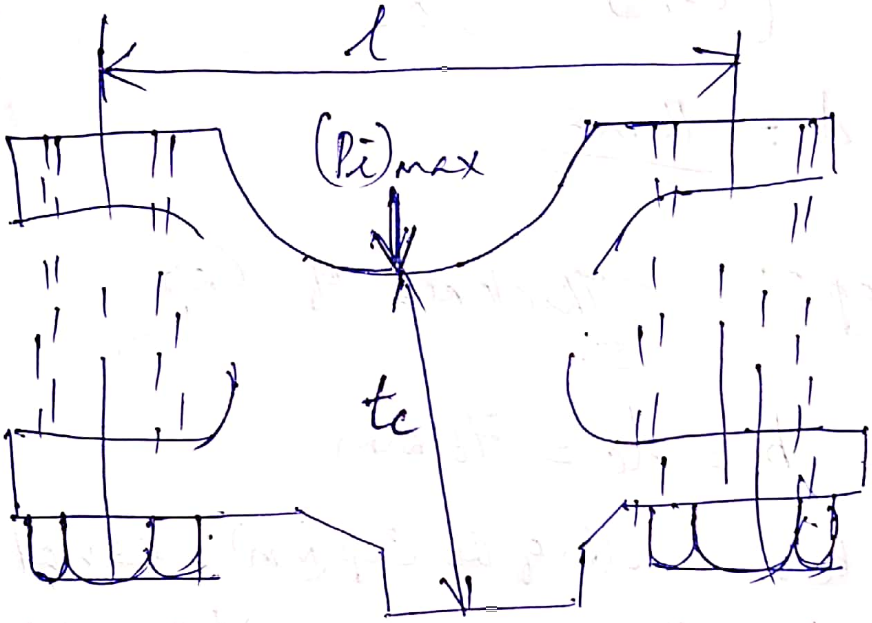
$$P_i = 9715.85 \text{ N}$$

Step 2 :- Diameter of Bolts



The Bolts are subjected to tensile force

d_c = Core dia of bolts (mm)



The Cap also called keep plate is subjected to inertia force $(P_i)_{max}$

t_c = Thickness of Big End Cap.

Since, there are 2 Bolts, each share the inertia force equally.

$$\therefore (P_i)_{max} = 2\left[\frac{\pi d_c^2}{4}\right]\sigma_t$$

σ_t = Permissible tensile stress for bolt mat. (N/mm^2)

$$(P_i)_{max} = 2\left[\frac{\pi d_c^2}{4}\right]\sigma_t$$

$$9715.35 = 2\left[\frac{3.14 \times d_c^2}{4}\right] 60$$

$d_c^2 = 103.09$ mm

$d_c = 10.15$ mm

$\boxed{d_c = \text{Core dia of bolt (mm)} \quad \cancel{d = \text{Nominal dia of bolt}}}$

$$d = \left[\frac{d_c}{0.8}\right] = \frac{10.15}{0.8} = 12.69 \text{ (or) } \underline{16 \text{ mm}}$$

$$d = \underline{16 \text{ mm}}$$

Step 3: **Thickness of Cap**

$b_c = l_c = 76$ mm

b_c = width of the cap (mm); equal to length of crank pin (or) big end bearing (l_c)

Also,

~~l = Length of Stroke~~ $\boxed{l = \text{Distance between the bolt / Span Length}}$

l = (Diameter of Crank Pin) + 2 (Thick of Bush) + Nominal dia of bolt (d) + Clearance (3 mm)

$$l = 58 + 2(3) + 16 + 3 = \underline{83 \text{ mm}}$$

$\boxed{d_c = 58\text{ mm}, \text{ Thick of bush} = 3\text{ mm}, \; d = 16\text{ mm and clearance} = 3\text{ mm}}$

$$M_b = \frac{(P_i)_{max} \cdot l}{6} = \frac{Wl}{6}$$

where M_b = Bending moment acting on cap (N-mm)

l = Span length (or) distance between the bolt centres (mm)

The cap is treated as a beam freely supported at the bolt centres and loaded in a manner intermediate between uniformly distributed and centrally concentrated load in which case the bending moment is $\frac{Wl}{6}$

$$\therefore M_b = \frac{(P_i)_{max}\, l}{6} = \frac{(9715.85)(83)}{6}$$

$$\boxed{M_b = 134402.59 \text{ N-mm}}$$

Thickness of the cap (t_c) is obtained by

$$\sigma_b = \frac{M_b y}{I}, \quad \text{where } I = \left[\frac{(b_c)(t_c)^3}{12}\right]$$

$$\boxed{\text{and } y = \frac{t_c}{2}}$$

$$I = \frac{(b_c)(t_c)^3}{12} = \left[\frac{(76)(t_c)^3}{12}\right] = 6.33\, t_c^3$$

$$y = \frac{t_c}{2}$$

Substituting the above expression in

$$\sigma_b = \frac{M_b\, y}{I} \quad (\text{OR}) \quad 80 = \frac{(134402.59)}{6.33\, t_c^3}\left(\frac{t_c}{2}\right)$$

$$t_c^2 = 132.7$$

$$\therefore t_c = 11.52\,\text{mm} \quad (\text{OR}) \quad \underline{\underline{12.0\,\text{mm}}}$$

28/04/2020

Problem No: 1 Determine the Dimensions of cross-section of the Connecting rod for a diesel engine with the following data:

Given Data:

Cylinder Bore (D) = 100 mm

length of Connecting rod (L) = 350 mm

Max gas pressure (P_{max}) = 4 MPa = 4 N/mm^2

Factor of safety (fs) = 6 [from PSG Data Book it is 3 to 6 Pg 7.122]

Step: I Force acting on the Connecting rod (CR)

Load due to gas (or) steam pressure

$$P_c = \left[\frac{\pi D^2}{4}\right] P_{max} = \left[\frac{\pi (100)^2}{4}\right](4) = 31415.9\ N$$

↳ Refer Pg no: 7.122 of PSG DD Book

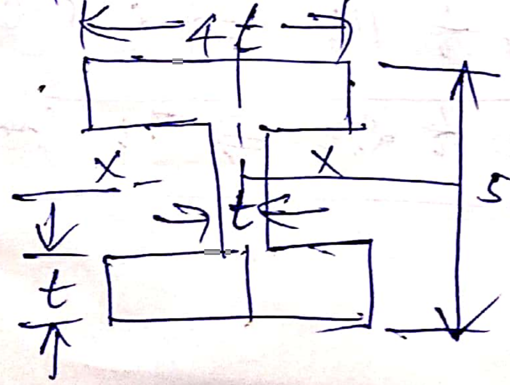
Step: II critical Buckling load (P_{cr})

$P_{cr} = P_c(fs)$ (Not available in PSG Data Book)

$$P_{cr} = 31415.9 \times 6 = 188495.58\ N$$

Step: III Calculation of 't'

From PSG Data Book Pg no 7.122



$A = 11t^2$

$K_{xx}^2 = 3.18t^2$

$K_{xx} = 1.78t$

The dimensions of Cross Section are calculated by applying Rankine's formula for buckling of the connecting rod in the Plane of rotation (or) about the XX axis. Acc to this formula.

$$P_{cr} = \frac{\sigma_c A}{1 + a\left[\frac{L}{K_{xx}}\right]^2}$$

⊛⊛ Ref Pg no 6.8 of PSG Data Book

where,

P_{cr} = Critical buckling load (N)

σ_c = Compressive yield stress (N/mm²)

A = Cross sec area of Connecting rod (mm²)

a = Constant depending upon material and end fixity co-efficient

$\boxed{a = \frac{1}{7500}}$ ⇒ for steel material
Not available in PSG Data

L = Length of connecting rod (mm)

K_{xx} = Radius of gyration (mm)

⊛ For connecting rod made of mild steel and Plain carbon steel $\sigma_c = 330$ N/mm²

From Pg no 1.17 & 37Mn2 (440 N/mm²) / 35 Mn2 Mo28 (540 N/mm²) / 35 Mn2 Mo48 (600 N/mm²)

⊛⊛ From Pg no ~~1.70~~ 1.10 ⇒ C40 ⇒ ~~380 N/mm²~~ ~~(Pg 1.12)~~
Pg 1.9 ⇒ 330 N/mm²

Calculated $P_{cr} = 188495.58$ N.

$$\therefore \; 188495.58 = \frac{(330)(11 \cdot t^2)}{1 + \frac{1}{7500}\left(\frac{350}{1.78t}\right)^2}$$

$$\frac{188495.58}{330 \times 11} = \frac{t^2}{1 + \frac{5.16}{t^2}}$$

$$51.93 = \frac{t^4}{t^2 + 5.16}$$

$$t^4 - 51.93t^2 - 267.96 = 0$$

The above expression is a quadratic eqn in (t^2)

$$t^2 = \frac{51.93 \pm \sqrt{(51.93)^2 + 4\,267.96}}{2}$$

$$= \frac{51.93 \pm 61.93}{2}$$

$$t^2 = 56.66$$

$$\therefore t = 7.53 \sim \underline{8.0 \text{ mm}}$$

Step IV: Dimensions of cross-section

$$B = 4t = 4(8) = 32 \text{ mm}$$

$$H = 5t = 5(8) = 40 \text{ mm}$$

Thickness of web = t = 8 mm

The width ($B = 32$ mm) is kept constant throughout the length of connecting rod.

Step: V Variation of height at the middle section

$H = 5t = 5 \times 8 = 40$ mm

At the small end (H_1) $= 0.85 H$

$= 0.85 \times 40 = 34$ mm.

At the big end (H_2) $= 1.2 H$

$= 1.2 \times 40 = 48$ mm

Dimension (B/H) of section at big end

$= 32$ mm $\times$ 48 mm.

Dim (B/H) of section at middle

$= 32$ mm $\times$ 40 mm

Dim (B/H) of section at small end

$= 32 \times 34$ mm.

Connecting rod Ends; Pg no 278 Shadwarayamoorthy

~~Small~~ (SMALL End) (OR) (Piston End) ⇒ Connected to Gudgeon Pin (OR) Cross head which is Reciprocating member

Big End (OR) (Crank End) ⇒ Connected to Crank Pin which is Rotating member

Load on Small End

The relative motion between the Pin and brg is subjected to IMPACT Loads due to pressure rise in the cylinder. Small end (Looks like) is made in the form of eye and brass bushes are provided for the bearings.

1. Big End brg of the C. Rod is generally SPLIT and the Cap is connected by means of BOLTS.
2. This enables easy dismantling of Connecting rod without removing crank shaft.
3. Big end of C. Rod is pressure lubricated by a hole in the crank.
4. The bushes of big end are split and held in position by a cap.

(5) Design of small end and big end of C. rod is based on the Permissible Bearing Pressure

(6) The bolts on the big ends are subjected to only tensile stress due to the Inertia of reciprocating Parts.

(7) They may be taken in proportion to Crank Pin and checked for tensile stress due to Inertia of the reciprocating parts.

(8) Length of connecting rod (l) depends upon the ratio of l/r

where r = Radius of Crank.

(9) C. Rod Consist of
- long Shank
- Small End
- Big End

(10) The Cross. Sec of the shank may be
(1) Rectangular
(2) Circular
(3) Tubular
(4) I-Section / H-Sec

(11) For low speed Engine → Circular Sec is Preferred,
High " " → I Sec is Preferred.

C. Parts for Diesel engine only :

1. Fuel pump. 2. Injector.

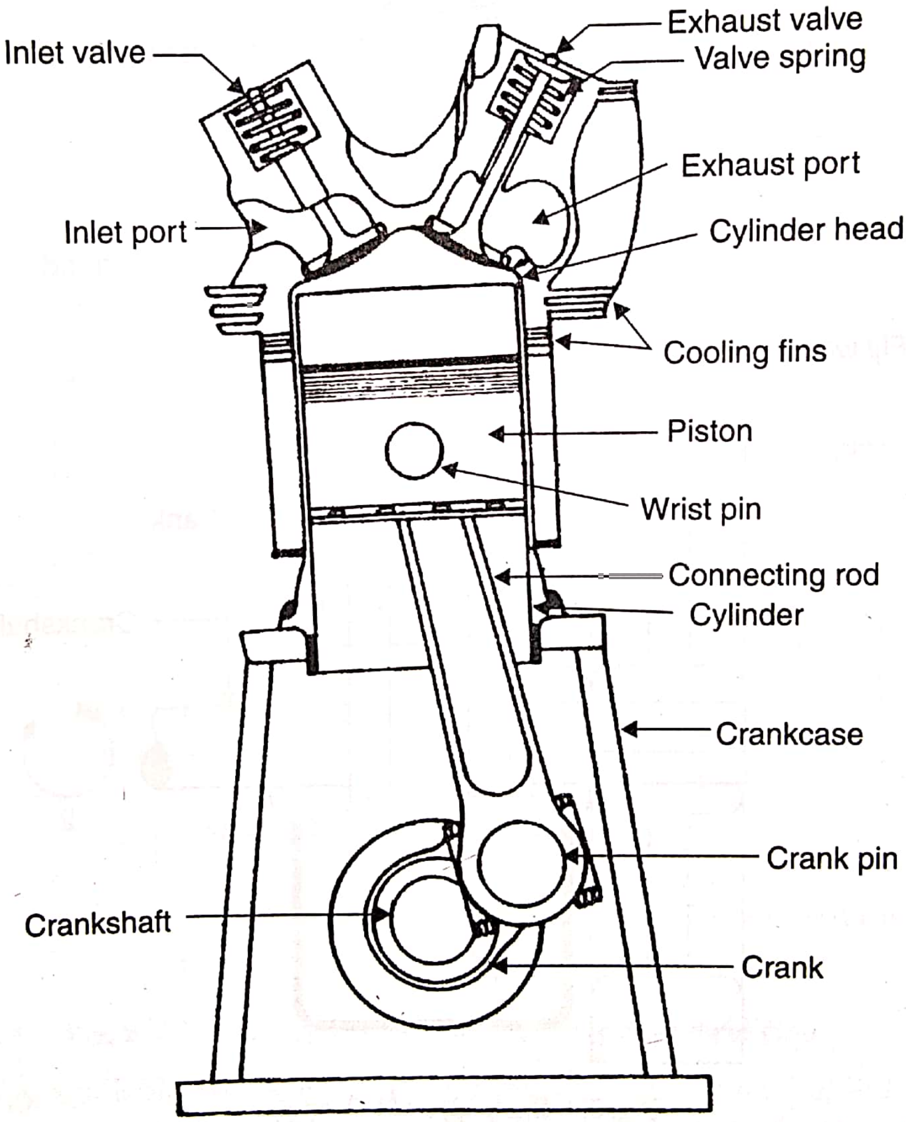


Fig. 2.5. Air-cooled I.C. engine.

It is made hollow for lightness since it is a reciprocating part.

6. **Connecting rod :**

Refer to Fig. 2.14. The connecting rod transmits the piston load to the crank, causing the latter to turn, thus converting the reciprocating motion of the piston into a rotary motion of the crankshaft. The lower or "big end" of the connecting rod turns on "crank pins".

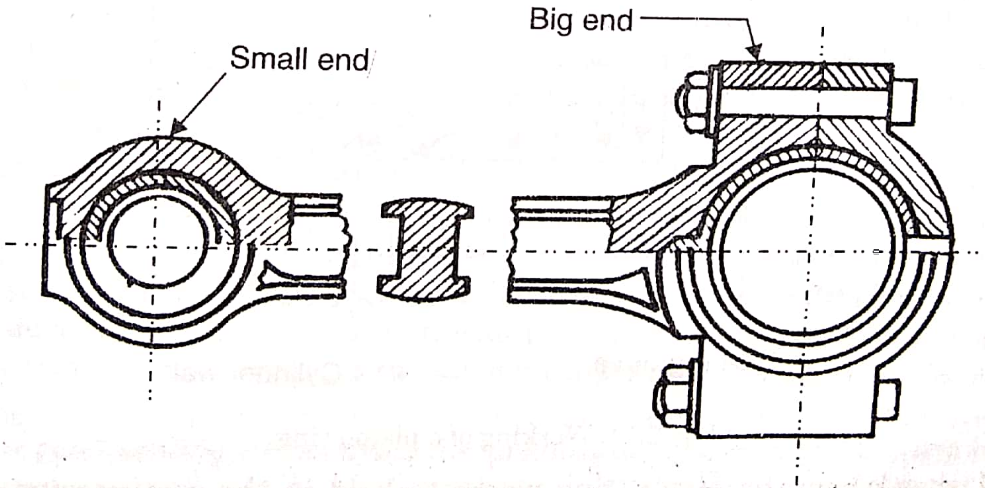


Fig. 2.14. Connecting rod.

The connecting rods are made of nickle, chrome and chrome vandium steels. For small engines the material may be aluminium.

11

Machine Parts

11·1. Connecting Rods

Connecting rod is a machine member used to transmit power from a reciprocating member to a rotary one or

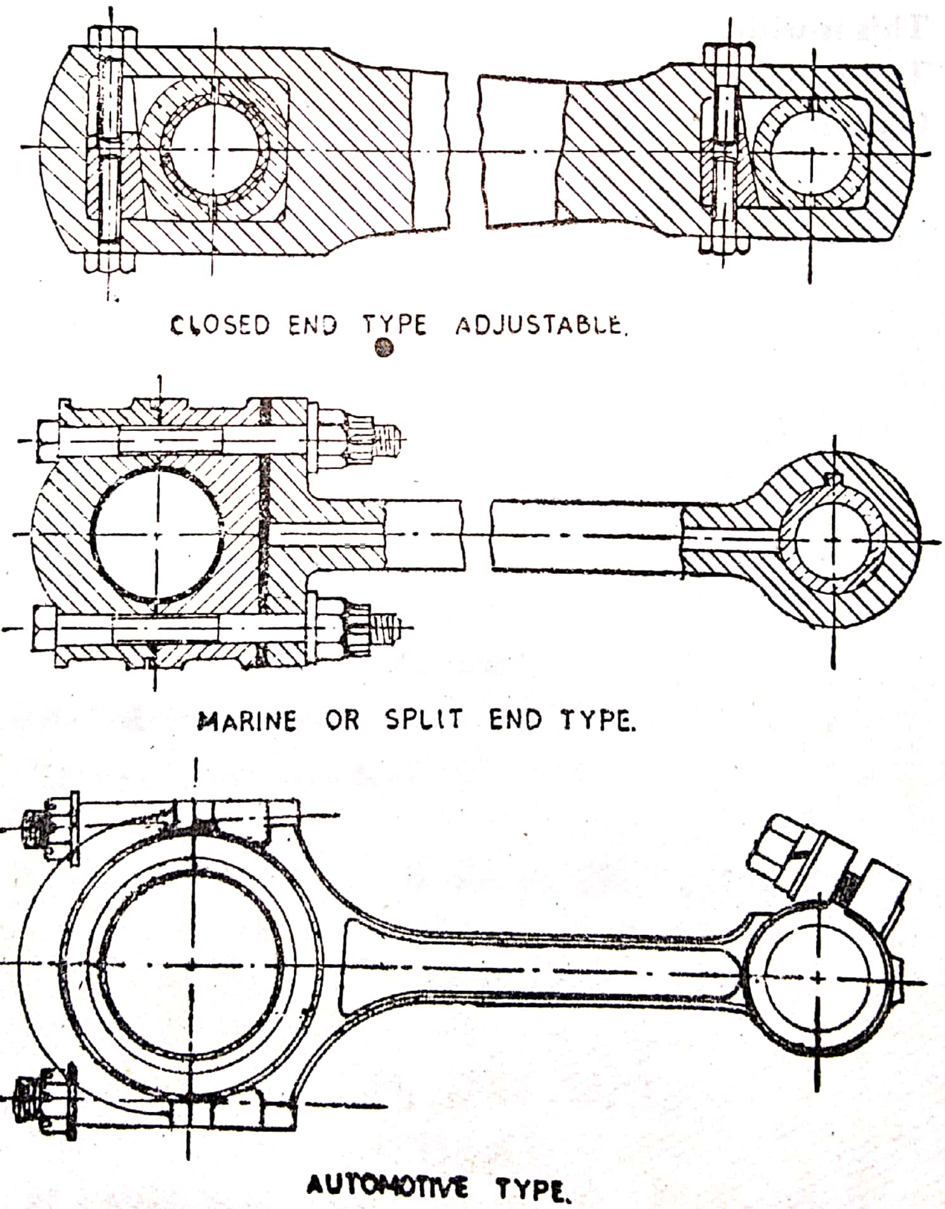


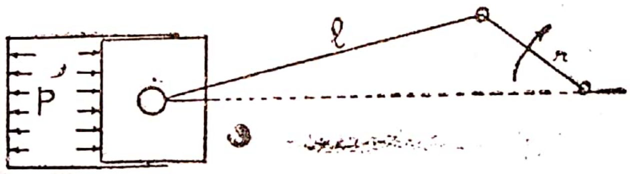
Fig. 11 1. Types of connecting rods.

vice versa. In this case of I.C. engines and steam engines, power from piston is transmitted to the crankshaft and in the case of reciprocating compressors, pumps, vibratory conveyors, power hammers and shaping machines, power is transmitted to the reciprocating member from crankshafts through connecting rods. A few types of connecting rods are shown in Fig. 11·1.

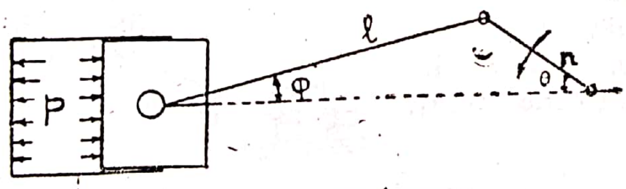
Generally connecting rods are subjected to fatigue due to alternating of fluctuating loads. Hence they should be manufactured from a material which will have good fatigue and shock resistance. Whereas mild steel and alloys of aluminium are used for light duty, alloy steels of Molybdenum and Chromium are used for heavy duty. To increase the fatigue resistance connecting rods are manufactured by forging.

11·2. Forces on Connecting Rods

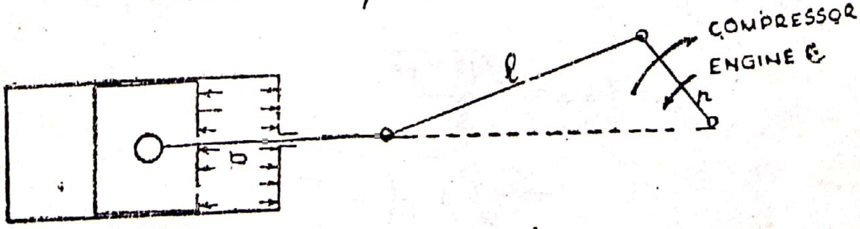
Connecting rods are subjected to compressive loads due to the fluid pressure on the piston or load coming on the reciprocating member [Fig. 11·2 (*a*) and 11·2 (*b*)]. In the case of double acting engines, compressors or pumps, connecting rods are subjected to alternate compression and tension [Fig. 11·2 (*c*)].



a) ENGINE. (SINGLE ACTING.)



b) COMPRESSOR / PUMP.



C) DOUBLE ACTING. (ENGINE/COMPRESSOR)

Fig. 11·2.

Let F_p be the force acting on the piston/reciprocating member

32.13 Connecting Rod

The connecting rod is the intermediate member between the piston and the crankshaft. Its primary function is to transmit the push and pull from the piston pin to the crankpin and thus convert the reciprocating motion of the piston into the rotary motion of the crank. The usual form of the connecting rod in internal combustion engines is shown in Fig. 32.9. It consists of a long shank, a small end and a big end. The cross-section of the shank may be rectangular, circular, tubular, *I*-section or *H*-section. Generally circular section is used for low speed engines while *I*-section is preferred for high speed engines.

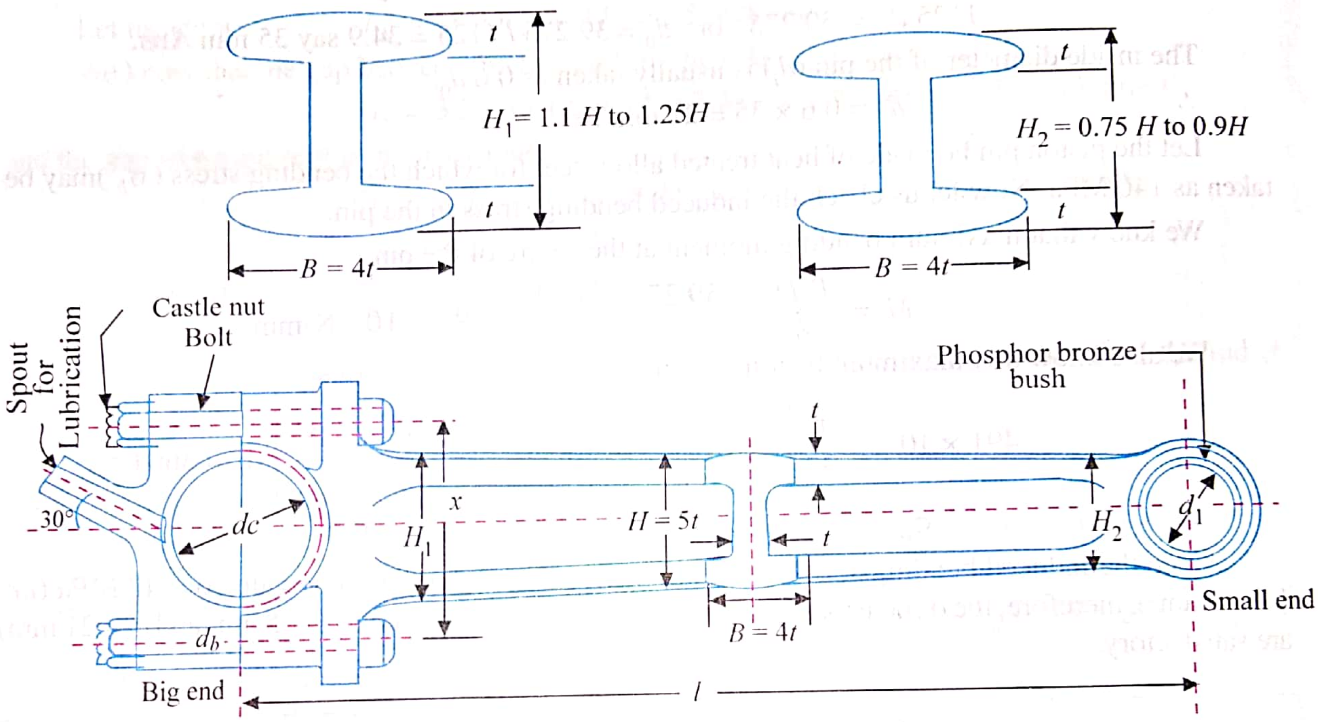


Fig. 32.9. Connecting rod.

The *length of the connecting rod (l) depends upon the ratio of l / r, where r is the radius of crank. It may be noted that the smaller length will decrease the ratio l / r. This increases the angularity of the connecting rod which increases the side thrust of the piston against the cylinder liner which in turn increases the wear of the liner. The larger length of the connecting rod will increase the ratio l / r. This decreases the angularity of the connecting rod and thus decreases the side thrust and the resulting wear of the cylinder. But the larger length of the connecting rod increases the overall height of the engine. Hence, a compromise is made and the ratio l / r is generally kept as 4 to 5.

The small end of the connecting rod is usually made in the form of an eye and is provided with a bush of phosphor bronze. It is connected to the piston by means of a piston pin.

The big end of the connecting rod is usually made split (in two **halves) so that it can be mounted easily on the crankpin bearing shells. The split cap is fastened to the big end with two cap bolts. The bearing shells of the big end are made of steel, brass or bronze with a thin lining (about 0.75 mm) of white metal or babbit metal. The wear of the big end bearing is allowed for by inserting thin metallic strips (known as ***shims***) about 0.04 mm thick between the cap and the fixed half of the connecting rod. As the wear takes place, one or more strips are removed and the bearing is trued up.

* It is the distance between the centres of small end and big end of the connecting rod.

** One half is fixed with the connecting rod and the other half (known as cap) is fastened with two cap bolts.

The connecting rods are usually manufactured by drop forging process and it should have adequate strength, stiffness and minimum weight. The material mostly used for connecting rods varies from mild carbon steels (having 0.35 to 0.45 percent carbon) to alloy steels (chrome-nickel or chrome-molybdenum steels). The carbon steel having 0.35 percent carbon has an ultimate tensile strength of about 650 MPa when properly heat treated and a carbon steel with 0.45 percent carbon has a ultimate tensile strength of 750 MPa. These steels are used for connecting rods of industrial engines. The alloy steels have an ultimate tensile strength of about 1050 MPa and are used for connecting rods of aeroengines and automobile engines.

The bearings at the two ends of the connecting rod are either splash lubricated or pressure lubricated. The big end bearing is usually splash lubricated while the small end bearing is pressure lubricated. In the **splash lubrication system**, the cap at the big end is provided with a dipper or spout and set at an angle in such a way that when the connecting rod moves downward, the spout will dip into the lubricating oil contained in the sump. The oil is forced up the spout and then to the big end bearing. Now when the connecting rod moves upward, a splash of oil is produced by the spout. This splashed up lubricant find its way into the small end bearing through the widely chamfered holes provided on the upper surface of the small end.

In the **pressure lubricating system,** the lubricating oil is fed under pressure to the big end bearing through the holes drilled in crankshaft, crankwebs and crank pin. From the big end bearing, the oil is fed to small end bearing through a fine hole drilled in the shank of the connecting rod. In some cases, the small end bearing is lubricated by the oil scrapped from the walls of the cyinder liner by the oil scraper rings.

32.14 Forces Acting on the Connecting Rod

The various forces acting on the connecting rod are as follows :

1. Force on the piston due to gas pressure and inertia of the reciprocating parts,
2. Force due to inertia of the connecting rod or inertia bending forces,
3. Force due to friction of the piston rings and of the piston, and
4. Force due to friction of the piston pin bearing and the crankpin bearing.

We shall now derive the expressions for the forces acting on a vertical engine, as discussed below.

1. *Force on the piston due to gas pressure and inertia of reciprocating parts*

Consider a connecting rod PC as shown in Fig. 32.10.

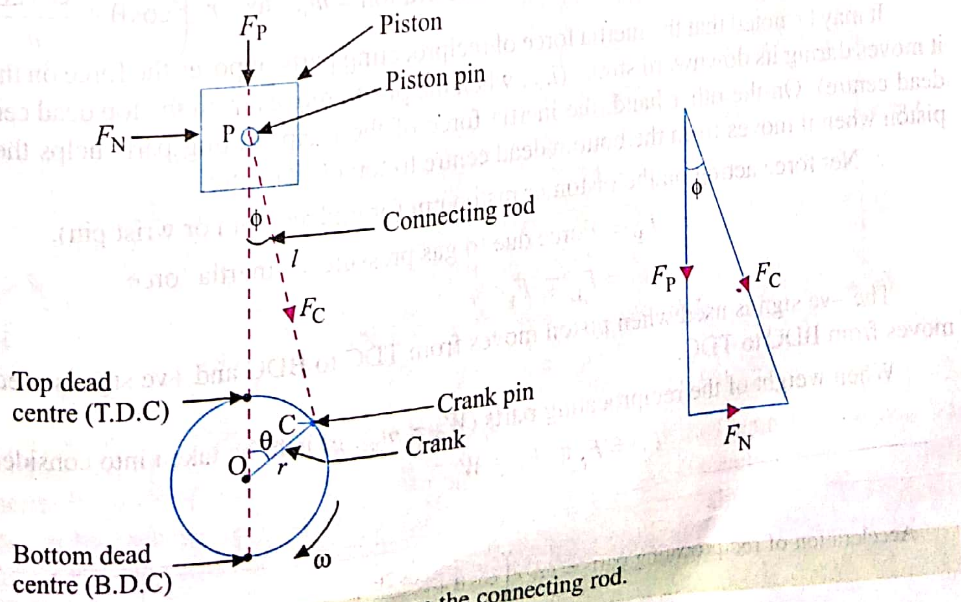


Fig. 32.10. Forces on the connecting rod.

due to inertia bending stress

32.15 Design of Connecting Rod

In designing a connecting rod, the following dimensions are required to be determined :

1. Dimensions of cross-section of the connecting rod,
2. Dimensions of the crankpin at the big end and the piston pin at the small end,
3. Size of bolts for securing the big end cap, and
4. Thickness of the big end cap.

The procedure adopted in determining the above mentioned dimensions is discussed as below :

2. Outside diameter of vessel = 1050 mm.

3. Wall thickness for cylindrical section = 18 mm.

4. Wall thickness for cover plate = 10 mm.

Problem 11.3 :

A steel tank having an internal diameter of 2.5 m is required to withstand an internal fluid pressure of 25 N/mm^2. The tensile and compressive elastic strength of the material is 200 N/mm^2. Find the thickness of the wall of the tank if the working value of the circumferential stress is two-third of the tensile elastic strength. (M.K. University)

Solution :

Internal diameter of tank, $d_i = 2.5$ m $= 2500$ mm.

The pressure of fluid, $p_i = 25$ N/mm^2

Tensile elastic strength of tank, $S_y = 200$ N/mm^2

Working stress, $S_t = \frac{2}{3} \times 200 = 133.3\ N/mm^2$

Now $\frac{S_t}{p_i} = \frac{133}{25} = 5.32$

Since $\frac{S_t}{p_i} < 6$, thick cylinder formula may be followed.

According to Lame's equation, the thickness of steel tank is given by

$$t = r_i\left[\left\{\frac{S_t}{S_t - 2p_i}\right\}^{1/2} - 1\right]$$ for mild steel material (ie, for duetile material)

Now, $r_i = \frac{d_i}{2} = \frac{2500}{2} = 1250$ mm.

$S_t = 133\ N/mm^2$

$p_i = 25\ N/mm^2$

Substituting in the above equation, we get,

$$t = 1250\left[\left(\frac{133}{133-(2\times 25)}\right)^{1/2} - 1\right] = 332 \text{ mm.}$$

The thickness of pressure vessel $= 332$ **mm. (Answer)**

Problem 11.4 :

Determine the wall thickness of a cylindrical vessel closed at both ends from the following data.

Internal pressure $= 20\ N/mm^2$

Internal diameter $= 300$ mm.

Allowable tensile stress $= 120\ N/mm^2$

Use the (a) maximum shear stress theory, and

(b) maximum strain theory. **(Madras University, April 2000)**

Solution :

Internal pressure of fluid, $p_i = 20\ N/mm^2$

Internal diameter of vessel, $d_i = 300$ mm.

Allowable tensile stress, $S_t = 120$ mm.

Let $t =$ Wall thickness of vessel.

(a) According to maximum shear stress theory (i.e., Lame's equation) the wall thickness is given by.

$$t = r_i \left[\left\{ \frac{S_t}{S_t - 2\,p_i} \right\}^{1/2} - 1 \right]$$

where r_i = Internal radius of vessel

$$= \frac{d_i}{2} = \frac{300}{2} = 150 \text{ mm.}$$

$$\therefore\ t = 150 \left[\left\{ \frac{120}{120 - 2 \times 20} \right\}^{1/2} - 1 \right] = \mathbf{34\ mm.}$$

(b) According to maximum strain theory (i.e., Clavarino's equation) the wall thickness is given by

$$t = r_i \left[\left\{ \frac{S_t + (1 - 2\,\mu)\,p_i}{S_t - (1 + \mu)\,p_i} \right\}^{1/2} - 1 \right]$$

where μ = Poisson's ratio = 0.3 (Assumed)

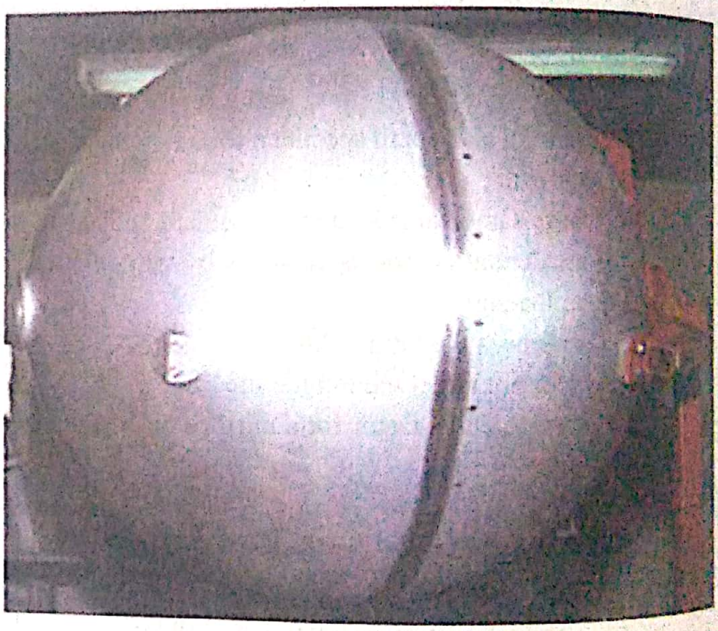
$$\therefore\ t = 150 \left[\left\{ \frac{120 + (1 - 2 \times 0.3)\,20}{120 - (1 + 0.3)\,20} \right\}^{1/2} - 1 \right] = \mathbf{25\ mm.}$$

Problem 11.5 :

CHAPTER

7

Pressure Vessels



7.1 Introduction

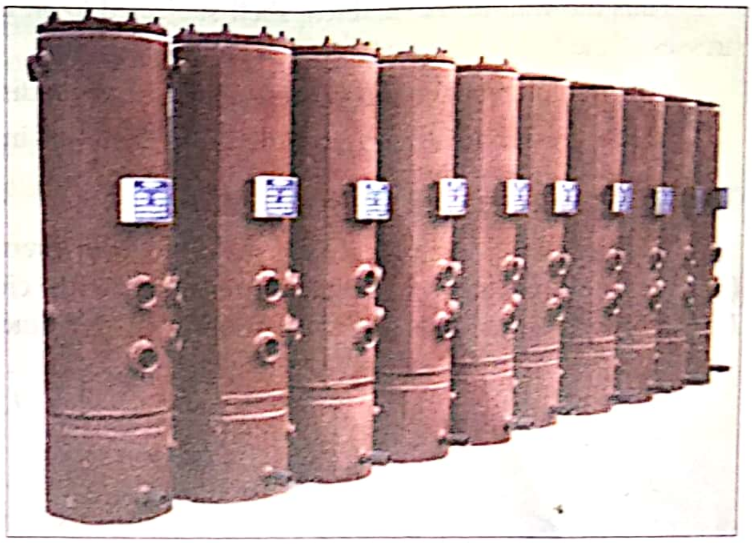
The pressure vessels (*i.e.* cylinders or tanks) are used to store fluids under pressure. The fluid being stored may undergo a change of state inside the pressure vessel as in case of steam boilers or it may combine with other reagents as in a chemical plant. The pressure vessels are designed with great care because rupture of a pressure vessel means an explosion which may cause loss of life and property. The material of pressure vessels may be brittle such as cast iron, or ductile such as mild steel.

7.2 Classification of Pressure Vessels

The pressure vessels may be classified as follows:

1. *According to the dimensions*. The pressure vessels, according to their dimensions, may be classified as ***thin shell*** or ***thick shell***. If the wall thickness of the shell (t) is less than 1/10 of the diameter of the shell (d), then it is called a ***thin shell***. On the other hand, if the wall thickness of the shell is greater than 1/10 of the diameter of the shell, then it is said to be a ***thick shell.*** Thin shells are used in boilers, tanks and pipes, whereas thick shells are used in high pressure cylinders, tanks, gun barrels etc.

Note: Another criterion to classify the pressure vessels as thin shell or thick shell is the internal fluid pressure (p) and the allowable stress (σ_t). If the internal fluid pressure (p) is less than 1/6 of the allowable stress, then it is called a ***thin shell.*** On the other hand, if the internal fluid pressure is greater than 1/6 of the allowable stress, then it is said to be a ***thick shell.***



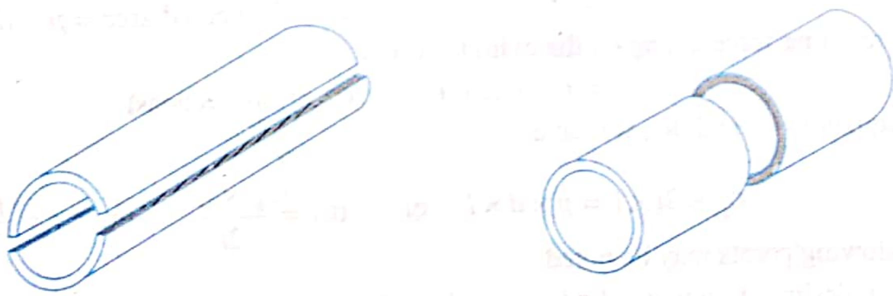
Pressure vessels.

2. ***According to the end construction.*** The pressure vessels, according to the end construction, may be classified as *open end* or *closed end*. A simple cylinder with a piston, such as cylinder of a press is an example of an open end vessel, whereas a tank is an example of a closed end vessel. In case of vessels having open ends, the circumferential or hoop stresses are induced by the fluid pressure, whereas in case of closed ends, longitudinal stresses in addition to circumferential stresses are induced.

7.3 Stresses in a Thin Cylindrical Shell due to an Internal Pressure

The analysis of stresses induced in a thin cylindrical shell are made on the following assumptions:

1. The effect of curvature of the cylinder wall is neglected.
2. The tensile stresses are uniformly distributed over the section of the walls.
3. The effect of the restraining action of the heads at the end of the pressure vessel is neglected.



(*a*) Failure of a cylindrical shell along the longitudinal section.

(*b*) Failure of a cylindrical shell along the transverse section.

Fig. 7.1. Failure of a cylindrical shell.

When a thin cylindrical shell is subjected to an internal pressure, it is likely to fail in the following two ways:

1. It may fail along the longitudinal section (*i.e.* circumferentially) splitting the cylinder into two troughs, as shown in Fig. 7.1 (*a*).
2. It may fail across the transverse section (*i.e.* longitudinally) splitting the cylinder into two cylindrical shells, as shown in Fig. 7.1 (*b*).

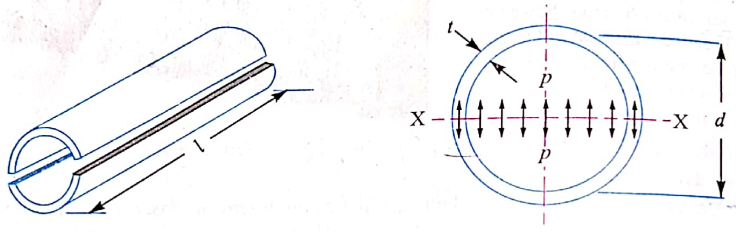
Thus the wall of a cylindrical shell subjected to an internal pressure has to withstand tensile stresses of the following two types:

(*a*) Circumferential or hoop stress, and (*b*) Longitudinal stress.

These stresses are discussed, in detail, in the following articles.

7.4 Circumferential or Hoop Stress

Consider a thin cylindrical shell subjected to an internal pressure as shown in Fig. 7.2 (*a*) and (*b*). A tensile stress acting in a direction tangential to the circumference is called ***circumferential*** or ***hoop stress***. In other words, it is a tensile stress on *longitudinal section (or on the cylindrical walls).



(*a*) View of shell. (*b*) Cross-section of shell.

Fig. 7.2. Circumferential or hoop stress.

Let p = Intensity of internal pressure,

d = Internal diameter of the cylindrical shell,

l = Length of the cylindrical shell,

t = Thickness of the cylindrical shell, and

σ_{t1} = Circumferential or hoop stress for the material of the cylindrical shell.

We know that the total force acting on a longitudinal section (*i.e.* along the diameter *X-X*) of the shell

$$= \text{Intensity of pressure} \times \text{Projected area} = p \times d \times l \qquad \ldots(i)$$

and the total resisting force acting on the cylinder walls

$$= \sigma_{t1} \times 2t \times l \qquad \ldots(\because \text{ of two sections}) \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), we have

$$\sigma_{t1} \times 2t \times l = p \times d \times l \quad \text{or} \quad \sigma_{t1} = \frac{p \times d}{2t} \quad \text{or} \quad t = \frac{p \times d}{2\,\sigma_{t1}} \qquad \ldots(iii)$$

The following points may be noted:

1. In the design of engine cylinders, a value of 6 mm to 12 mm is added in equation (*iii*) to permit reboring after wear has taken place. Therefore

$$t = \frac{p \times d}{2\,\sigma_{t1}} + 6 \text{ to } 12 \text{ mm}$$

2. In constructing large pressure vessels like steam boilers, riveted joints or welded joints are used in joining together the ends of steel plates. In case of riveted joints, the wall thickness of the cylinder,

$$t = \frac{p \times d}{2\sigma_{t1} \times \eta_l}$$

where η_l = Efficiency of the longitudinal riveted joint.

* A section cut from a cylinder by a plane that contains the axis is called longitudinal section.

3. In case of cylinders of ductile material, the value of circumferential stress (σ_{t1}) may be taken 0.8 times the yield point stress (σ_y) and for brittle materials, σ_{t1} may be taken as 0.125 times the ultimate tensile stress (σ_u).
4. In designing steam boilers, the wall thickness calculated by the above equation may be compared with the minimum plate thickness as provided in boiler code as given in the following table.

Table 7.1. Minimum plate thickness for steam boilers.

Boiler diameter	*Minimum plate thickness (t)*
0.9 m or less	6 mm
Above 0.9 m and upto 1.35 m	7.5 mm
Above 1.35 m and upto 1.8 m	9 mm
Over 1.8 m	12 mm

Note: If the calculated value of t is less than the code requirement, then the latter should be taken, otherwise the calculated value may be used.

The boiler code also provides that the factor of safety shall be at least 5 and the steel of the plates and rivets shall have as a minimum the following ultimate stresses.

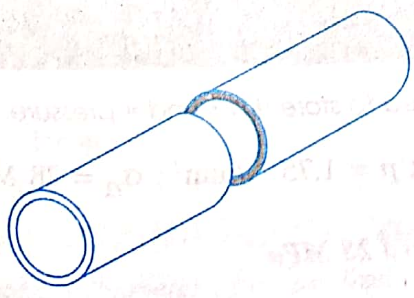
Tensile stress, $\sigma_t = 385$ MPa

Compressive stress, $\sigma_c = 665$ MPa

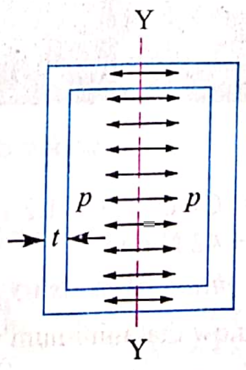
Shear stress, $\tau = 308$ MPa

7.5 Longitudinal Stress

Consider a closed thin cylindrical shell subjected to an internal pressure as shown in Fig. 7.3 (*a*) and (*b*). A tensile stress acting in the direction of the axis is called ***longitudinal stress***. In other words, it is a tensile stress acting on the *transverse or circumferential section *Y-Y* (or on the ends of the vessel).



(*a*) View of shell.



(*b*) Cross-section of shell.

Fig. 7.3. Longitudinal stress.

Let σ_{t2} = Longitudinal stress.

In this case, the total force acting on the transverse section (*i.e.* along *Y-Y*)

$$= \text{Intensity of pressure} \times \text{Cross-sectional area}$$

$$= p \times \frac{\pi}{4}(d)^2 \qquad ...(i)$$

and total resisting force

$$= \sigma_{t2} \times \pi\, d.t \qquad ...(ii)$$

* A section cut from a cylinder by a plane at right angles to the axis of the cylinder is called transverse section.

From equations (*i*) and (*ii*), we have

$$\sigma_{t2} \times \pi\, d.t = p \times \frac{\pi}{4}(d)^2$$

$$\therefore \qquad \sigma_{t2} = \frac{p \times d}{4\,t} \quad \text{or} \quad t = \frac{p \times d}{4\,\sigma_{t2}}$$

If η_c is the efficiency of the circumferential joint, then

$$t = \frac{p \times d}{4\sigma_{t2} \times \eta_c}$$

From above we see that the longitudinal stress is half of the circumferential or hoop stress. Therefore, the design of a pressure vessel must be based on the maximum stress *i.e.* hoop stress.

Example 7.1. *A thin cylindrical pressure vessel of 1.2 m diameter generates steam at a pressure of 1.75 N/mm². Find the minimum wall thickness, if (a) the longitudinal stress does not exceed 28 MPa; and (b) the circumferential stress does not exceed 42 MPa.*

Cylinders and tanks are used to store fluids under pressure.

Solution. Given : d = 1.2 m = 1200 mm ; p = 1.75 N/mm² ; σ_{t2} = 28 MPa = 28 N/mm² ; σ_{t1} = 42 MPa = 42 N/mm²

(a) When longitudinal stress (σ_{t2}) does not exceed 28 MPa

We know that minimum wall thickness,

$$t = \frac{p\,.\,d}{4\,\sigma_{t2}} = \frac{1.75 \times 1200}{4 \times 28} = 18.75 \text{ say } 20 \text{ mm} \quad \textbf{Ans.}$$

(b) When circumferential stress (σ_{t1}) does not exceed 42 MPa

We know that minimum wall thickness,

$$t = \frac{p\,.\,d}{2\,\sigma_{t1}} = \frac{1.75 \times 1200}{2 \times 42} = 25 \text{ mm} \quad \textbf{Ans.}$$

Example 7.2. *A thin cylindrical pressure vessel of 500 mm diameter is subjected to an internal pressure of 2 N/mm². If the thickness of the vessel is 20 mm, find the hoop stress, longitudinal stress and the maximum shear stress.*

Solution. Given : d = 500 mm ; p = 2 N/mm² ; t = 20 mm

Hoop stress

We know that hoop stress,

$$\sigma_{t1} = \frac{p \cdot d}{2\,t} = \frac{2 \times 500}{2 \times 20} = 25 \text{ N/mm}^2 = 25 \text{ MPa } \textbf{Ans.}$$

Longitudinal stress

We know that longitudinal stress,

$$\sigma_{t2} = \frac{p \cdot d}{4\,t} = \frac{2 \times 500}{4 \times 20} = 12.5 \text{ N/mm}^2 = 12.5 \text{ MPa } \textbf{Ans.}$$

Maximum shear stress

We know that according to maximum shear stress theory, the maximum shear stress is one-half the algebraic difference of the maximum and minimum principal stress. Since the maximum principal stress is the hoop stress (σ_{t1}) and minimum principal stress is the longitudinal stress (σ_{t2}), therefore maximum shear stress,

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{25 - 12.5}{2} = 6.25 \text{ N/mm}^2 = 6.25 \text{ MPa } \textbf{Ans.}$$

Example 7.3. *An hydraulic control for a straight line motion, as shown in Fig. 7.4, utilises a spherical pressure tank 'A' connected to a working cylinder B. The pump maintains a pressure of 3 N/mm² in the tank.*

1. If the diameter of pressure tank is 800 mm, determine its thickness for 100% efficiency of the joint. Assume the allowable tensile stress as 50 MPa.

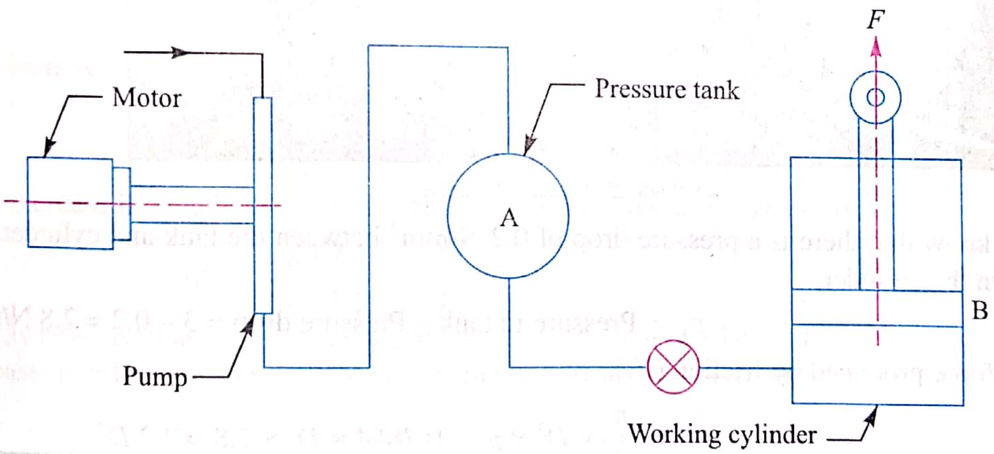


Fig. 7.4

2. Determine the diameter of a cast iron cylinder and its thickness to produce an operating force F = 25 kN. Assume (i) an allowance of 10 per cent of operating force F for friction in the cylinder and packing, and (ii) a pressure drop of 0.2 N/mm² between the tank and cylinder. Take safe stress for cast iron as 30 MPa.

3. Determine the power output of the cylinder, if the stroke of the piston is 450 mm and the time required for the working stroke is 5 seconds.

4. Find the power of the motor, if the working cycle repeats after every 30 seconds and the efficiency of the hydraulic control is 80 percent and that of pump 60 percent.

Solution. Given : $p = 3$ N/mm² ; $d = 800$ mm ; $\eta = 100\% = 1$; $\sigma_{t1} = 50$ MPa $= 50$ N/mm² ; $F = 25$ kN $= 25 \times 10^3$ N ; $\sigma_{tc} = 30$ MPa $= 30$ N/mm² : $\eta_H = 80\% = 0.8$; $\eta_P = 60\% = 0.6$

1. Thickness of pressure tank

We know that thickness of pressure tank,

$$t = \frac{p \cdot d}{2\sigma_{t1} \cdot \eta} = \frac{3 \times 800}{2 \times 50 \times 1} = 24 \text{ mm} \quad \textbf{Ans.}$$

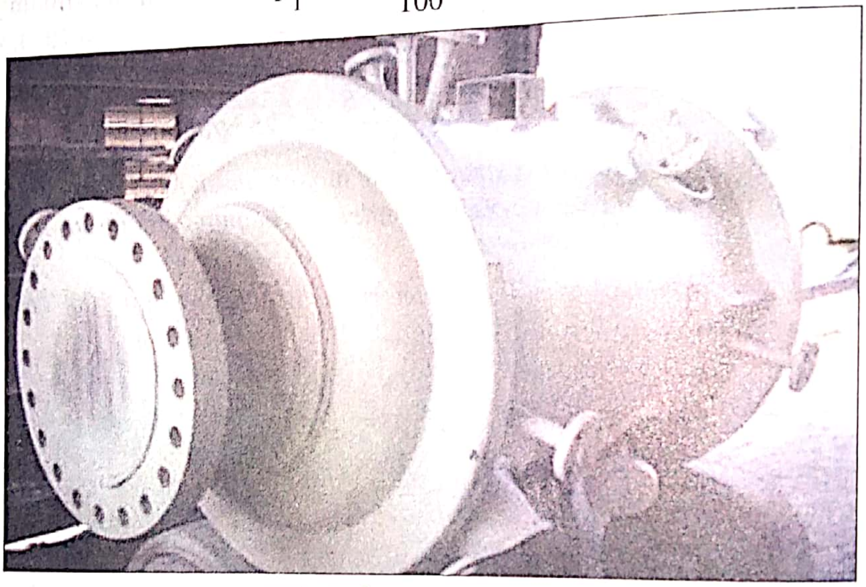
2. Diameter and thickness of cylinder

Let D = Diameter of cylinder, and

t_1 = Thickness of cylinder.

Since an allowance of 10 per cent of operating force F is provided for friction in the cylinder and packing, therefore total force to be produced by friction,

$$F_1 = F + \frac{10}{100} F = 1.1\, F = 1.1 \times 25 \times 10^3 = 27\,500 \text{ N}$$



Jacketed pressure vessel.

We know that there is a pressure drop of 0.2 N/mm² between the tank and cylinder, therefore pressure in the cylinder,

p_1 = Pressure in tank – Pressure drop = 3 – 0.2 = 2.8 N/mm²

and total force produced by friction (F_1),

$$27\,500 = \frac{\pi}{4} \times D^2 \times p_1 = 0.7854 \times D^2 \times 2.8 = 2.2\, D^2$$

$\therefore \quad D^2 = 27\,500 / 2.2 = 12\,500 \quad$ or $\quad D = 112$ mm **Ans.**

We know that thickness of cylinder,

$$t_1 = \frac{p_1 \cdot D}{2\,\sigma_{tc}} = \frac{2.8 \times 112}{2 \times 30} = 5.2 \text{ mm} \quad \textbf{Ans.}$$

3. Power output of the cylinder

We know that stroke of the piston

= 450 mm = 0.45 m ...(Given)

and time required for working stroke

= 5 s ...(Given)

∴ Distance moved by the piston per second

$$= \frac{0.45}{5} = 0.09 \text{ m}$$

We know that work done per second

= Force × Distance moved per second

= 27 500 × 0.09 = 2475 N-m

∴ Power output of the cylinder

= 2475 W = 2.475 kW **Ans.** ...(∵ 1 N-m/s = 1 W)

4. Power of the motor

Since the working cycle repeats after every 30 seconds, therefore the power which is to be produced by the cylinder in 5 seconds is to be provided by the motor in 30 seconds.

∴ Power of the motor

$$= \frac{\text{Power of the cylinder}}{\eta_H \times \eta_P} \times \frac{5}{30} = \frac{2.475}{0.8 \times 0.6} \times \frac{5}{30} = 0.86 \text{ kW}$$ **Ans.**

7.6 Change in Dimensions of a Thin Cylindrical Shell due to an Internal Pressure

When a thin cylindrical shell is subjected to an internal pressure, there will be an increase in the diameter as well as the length of the shell.

Let l = Length of the cylindrical shell,

d = Diameter of the cylindrical shell,

t = Thickness of the cylindrical shell,

p = Intensity of internal pressure,

E = Young's modulus for the material of the cylindrical shell, and

μ = Poisson's ratio.

The increase in diameter of the shell due to an internal pressure is given by,

$$\delta d = \frac{p.d^2}{2\,t.E}\left(1 - \frac{\mu}{2}\right)$$

The increase in length of the shell due to an internal pressure is given by,

$$\delta l = \frac{p.d.l}{2\,t.E}\left(\frac{1}{2} - \mu\right)$$

It may be noted that the increase in diameter and length of the shell will also increase its volume. The increase in volume of the shell due to an internal pressure is given by

$$\delta V = \text{Final volume} - \text{Original volume} = \frac{\pi}{4}(d + \delta d)^2 (l + \delta l) - \frac{\pi}{4} \times d^2.l$$

$$= \frac{\pi}{4}(d^2.\delta l + 2\,d.l.\delta d)$$ ...(Neglecting small quantities)

Example 7.4. *Find the thickness for a tube of internal diameter 100 mm subjected to an internal pressure which is 5/8 of the value of the maximum permissible circumferential stress. Also find the increase in internal diameter of such a tube when the internal pressure is 90 N/mm². Take E = 205 kN/mm² and μ = 0.29. Neglect longitudinal strain.*

Solution. Given : $p = 5/8 \times \sigma_{t1} = 0.625\,\sigma_{t1}$; d = 100 mm ; p_1 = 90 N/mm² ; E = 205 kN/mm² = 205 × 10³ N/mm² ; μ = 0.29

Thickness of a tube

We know that thickness of a tube,

$$t = \frac{p.d}{2\,\sigma_{t1}} = \frac{0.625\,\sigma_{t1} \times 100}{2\,\sigma_{t1}} = 31.25 \text{ mm}$$ **Ans.**

Increase in diameter of a tube

We know that increase in diameter of a tube,

$$\delta d = \frac{p_1 d^2}{2\,t.E}\left(1 - \frac{\mu}{2}\right) = \frac{90\,(100)^2}{2 \times 31.25 \times 205 \times 10^3}\left[1 - \frac{0.29}{2}\right] \text{mm}$$

$$= 0.07\,(1 - 0.145) = 0.06 \text{ mm } \textbf{Ans.}$$

7.7 Thin Spherical Shells Subjected to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure as shown in Fig. 7.5.

Let V = Storage capacity of the shell,

p = Intensity of internal pressure,

d = Diameter of the shell,

t = Thickness of the shell,

σ_t = Permissible tensile stress for the shell material.

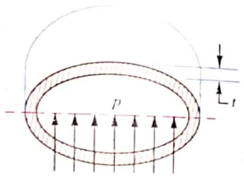


Fig. 7.5. Thin spherical shell.

In designing thin spherical shells, we have to determine 1. Diameter of the shell, and 2. Thickness of the shell.

1. *Diameter of the shell*

We know that the storage capacity of the shell,

$$V = \frac{4}{3} \times \pi\, r^3 = \frac{\pi}{6} \times d^3 \quad \text{or} \quad d = \left(\frac{6\,V}{\pi}\right)^{1/3}$$

2. *Thickness of the shell*

As a result of the internal pressure, the shell is likely to rupture along the centre of the sphere. Therefore force tending to rupture the shell along the centre of the sphere or bursting force,

$$= \text{Pressure} \times \text{Area} = p \times \frac{\pi}{4} \times d^2 \qquad \text{...(i)}$$

and resisting force of the shell

$$= \text{Stress} \times \text{Resisting area} = \sigma_t \times \pi\, d.t \qquad \text{...(ii)}$$

Equating equations (*i*) and (*ii*), we have

$$p \times \frac{\pi}{4} \times d^2 = \sigma_t \times \pi\, d.t$$

or

$$t = \frac{p.d}{4\,\sigma_t}$$

If η is the efficiency of the circumferential joints of the spherical shell, then

$$t = \frac{p.d}{4\,\sigma_t.\eta}$$

The Trans-Alaska Pipeline carries crude oil 1,284 kilometres through Alaska. The pipeline is 1.2 metres in diameter and can transport 318 million litres of crude oil a day.

Example 7.5. *A spherical vessel 3 metre diameter is subjected to an internal pressure of 1.5 N/mm². Find the thickness of the vessel required if the maximum stress is not to exceed 90 MPa. Take efficiency of the joint as 75%.*

Solution. Given: d = 3 m = 3000 mm ; p = 1.5 N/mm² ; σ_t = 90 MPa = 90 N/mm² ; η = 75% = 0.75

We know that thickness of the vessel,

$$t = \frac{p.d}{4\,\sigma_t\,.\,\eta} = \frac{1.5 \times 3000}{4 \times 90 \times 0.75} = 16.7 \text{ say } 18 \text{ mm } \textbf{Ans.}$$

7.8 Change in Dimensions of a Thin Spherical Shell due to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure as shown in Fig. 7.5.

Let d = Diameter of the spherical shell,

t = Thickness of the spherical shell,

p = Intensity of internal pressure,

E = Young's modulus for the material of the spherical shell, and

μ = Poisson's ratio.

Increase in diameter of the spherical shell due to an internal pressure is given by,

$$\delta d = \frac{p\,d^2}{4\,t.E}\,(1 - \mu) \qquad \text{...(i)}$$

and increase in volume of the spherical shell due to an internal pressure is given by,

$$\delta V = \text{Final volume} - \text{Original volume} = \frac{\pi}{6}\,(d + \delta d)^3 - \frac{\pi}{6} \times d^3$$

$$= \frac{\pi}{6}\,(3\,d^2 \times \delta d) \qquad \text{...(Neglecting higher terms)}$$

Substituting the value of δd from equation (*i*), we have

$$\delta V = \frac{3\,\pi\,d^2}{6}\left[\frac{p.d^2}{4\,t.E}\,(1 - \mu)\right] = \frac{\pi\,p\,d^4}{8\,t.E}\,(1 - \mu)$$

Example 7.6. *A seamless spherical shell, 900 mm in diameter and 10 mm thick is being filled with a fluid under pressure until its volume increases by 150 × 10³ mm³. Calculate the pressure exerted by the fluid on the shell, taking modulus of elasticity for the material of the shell as 200 kN/mm² and Poisson's ratio as 0.3.*

Solution. Given : d = 900 mm ; t = 10 mm ; δV = 150 × 10³ mm³ ; E = 200 kN/mm² = 200 × 10³ N/mm² ; μ = 0.3

Let p = Pressure exerted by the fluid on the shell.

We know that the increase in volume of the spherical shell (δV),

$$150 \times 10^3 = \frac{\pi\,p\,d^4}{8\,t\,E}\,(1 - \mu) = \frac{\pi\,p\,(900)^4}{8 \times 10 \times 200 \times 10^3}\,(1 - 0.3) = 90\,190\,p$$

$$\therefore \quad p = 150 \times 10^3 / 90\,190 = 1.66 \text{ N/mm}^2 \textbf{ Ans.}$$

7.9 Thick Cylindrical Shells Subjected to an Internal Pressure

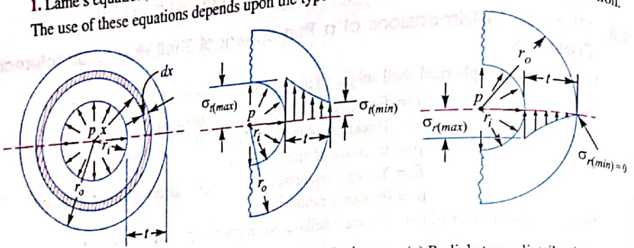
When a cylindrical shell of a pressure vessel, hydraulic cylinder, gunbarrel and a pipe is subjected to a very high internal fluid pressure, then the walls of the cylinder must be made extremely heavy or thick.

In thin cylindrical shells, we have assumed that the tensile stresses are uniformly distributed over the section of the walls. But in the case of thick wall cylinders as shown in Fig. 7.6 (*a*), the stress over the section of the walls cannot be assumed to be uniformly distributed. They develop both tangential and radial stresses with values which are dependent upon the radius of the element under consideration. The distribution of stress in a thick cylindrical shell is shown in Fig. 7.6 (*b*) and (*c*). We see that the tangential stress is maximum at the inner surface and minimum at the outer surface of the shell. The radial stress is maximum at the inner surface and zero at the outer surface of the shell.

In the design of thick cylindrical shells, the following equations are mostly used:

1. Lame's equation; 2. Birnie's equation; 3. Clavarino's equation; and 4. Barlow's equation.

The use of these equations depends upon the type of material used and the end construction.



(a) Thick cylindrical shell. (b) Tangential stress distribution. (c) Radial stress distribution.

Fig. 7.6. Stress distribution in thick cylindrical shells subjected to internal pressure.

Let r_o = Outer radius of cylindrical shell,

r_i = Inner radius of cylindrical shell,

t = Thickness of cylindrical shell = $r_o - r_i$,

p = Intensity of internal pressure,

μ = Poisson's ratio,

σ_t = Tangential stress, and

σ_r = Radial stress.

All the above mentioned equations are now discussed, in detail, as below:

1. *Lame's equation.* Assuming that the longitudinal fibres of the cylindrical shell are equally strained, Lame has shown that the tangential stress at any radius x is,

$$\sigma_t = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} + \frac{(r_i)^2 (r_o)^2}{x^2}\left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2}\right]$$

While designing a tanker, the pressure added by movement of the vehicle also should be considered.

and radial stress at any radius x,

$$\sigma_r = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} - \frac{(r_i)^2 (r_o)^2}{x^2}\left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2}\right]$$

Since we are concerned with the internal pressure ($p_i = p$) only, therefore substituting the value of external pressure, $p_o = 0$.

$\therefore$ Tangential stress at any radius x,

$$\sigma_t = \frac{p (r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right] \quad \text{...(i)}$$

and radial stress at any radius x,

$$\sigma_r = \frac{p (r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right] \quad \text{...(ii)}$$

We see that the tangential stress is always a tensile stress whereas the radial stress is a compressive stress. We know that the tangential stress is maximum at the inner surface of the shell (*i.e.* when $x = r_i$) and it is minimum at the outer surface of the shell (*i.e.* when $x = r_o$). Substituting the value of $x = r_i$ and $x = r_o$ in equation (*i*), we find that the *maximum tangential stress at the inner surface of the shell,

$$\sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

and minimum tangential stress at the outer surface of the shell,

$$\sigma_{t(min)} = \frac{2\,p\,(r_i)^2}{(r_o)^2 - (r_i)^2}$$

We also know that the radial stress is maximum at the inner surface of the shell and zero at the outer surface of the shell. Substituting the value of $x = r_i$ and $x = r_o$ in equation (*ii*), we find that maximum radial stress at the inner surface of the shell,

$$\sigma_{r(max)} = -p \text{ (compressive)}$$

and minimum radial stress at the outer surface of the shell,

$$\sigma_{r(min)} = 0$$

In designing a thick cylindrical shell of brittle material (*e.g.* cast iron, hard steel and cast aluminium) with closed or open ends and in accordance with the maximum normal stress theory failure, the tangential stress induced in the cylinder wall,

$$\sigma_t = \sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

Since $r_o = r_i + t$, therefore substituting this value of r_o in the above expression, we get

$$\sigma_t = \frac{p\,[(r_i + t)^2 + (r_i)^2]}{(r_i + t)^2 - (r_i)^2}$$

$$\sigma_t (r_i + t)^2 - \sigma_t (r_i)^2 = p\,(r_i + t)^2 + p\,(r_i)^2$$

$$(r_i + t)^2 (\sigma_t - p) = (r_i)^2 (\sigma_t + p)$$

$$\frac{(r_i + t)^2}{(r_i)^2} = \frac{\sigma_t + p}{\sigma_t - p}$$

* The maximum tangential stress is always greater than the internal pressure acting on the shell.

$$\frac{r_i + t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}} \quad \text{or} \quad 1 + \frac{t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}}$$

$$\therefore \quad \frac{t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \quad \text{or} \quad t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] \quad ...(iii)$$

The value of σ_t for brittle materials may be taken as 0.125 times the ultimate tensile strength (σ_u).

We have discussed above the design of a thick cylindrical shell of brittle materials. In case of cylinders made of ductile material, Lame's equation is modified according to maximum shear stress theory.

According to this theory, the maximum shear stress at any point in a strained body is equal to one-half the algebraic difference of the maximum and minimum principal stresses at that point. We know that for a thick cylindrical shell,

Maximum principal stress at the inner surface,

$$\sigma_{t\,(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

and minimum principal stress at the outer surface,

$$\sigma_{t(min)} = -p$$

$\therefore$ Maximum shear stress,

$$\tau = \tau_{max} = \frac{\sigma_{t(max)} - \sigma_{t(min)}}{2} = \frac{\dfrac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} - (-p)}{2}$$

$$= \frac{p\,[(r_o)^2 + (r_i)^2] + p\,[(r_o)^2 - (r_i)^2]}{2[(r_o)^2 - (r_i)^2]} = \frac{2\,p\,(r_o)^2}{2[(r_o)^2 - (r_i)^2]}$$

$$= \frac{p\,(r_i + t)^2}{(r_i + t)^2 - (r_i)^2} \quad ... (\because r_o = r_i + t)$$

or

$$\tau(r_i + t)^2 - \tau(r_i)^2 = p(r_i + t)^2$$

$$(r_i + t)^2\,(\tau - p) = \tau(r_i)^2$$

$$\frac{(r_i + t)^2}{(r_i)^2} = \frac{\tau}{\tau - p}$$

$$\frac{r_i + t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} \quad \text{or} \quad 1 + \frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}}$$

$$\therefore \quad \frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} - 1 \quad \text{or} \quad t = r_i \left[\sqrt{\frac{\tau}{\tau - p}} - 1\right] \quad ...(iv)$$

The value of shear stress (τ) is usually taken as one-half the tensile stress (σ_t). Therefore the above expression may be written as

$$t = r_i \left[\sqrt{\frac{\sigma_t}{\sigma_t - 2p}} - 1\right] \quad ...(v)$$

From the above expression, we see that if the internal pressure (p) is equal to or greater than the allowable working stress (σ_t or τ), then no thickness of the cylinder wall will prevent failure. Thus, it is impossible to design a cylinder to withstand fluid pressure greater than the allowable working stress for a given material. This difficulty is overcome by using compound cylinders (See Art. 7.10).

2. *Birnie's equation.* In case of open-end cylinders (such as pump cylinders, rams, gun barrels etc.) made of ductile material (*i.e.* low carbon steel, brass, bronze, and aluminium alloys), the allowable stresses cannot be determined by means of maximum-stress theory of failure. In such cases, the maximum-strain theory is used. According to this theory, the failure occurs when the strain reaches a limiting value and Birnie's equation for the wall thickness of a cylinder is

$$t = r_i \left[\sqrt{\frac{\sigma_t + (1 - \mu)\,p}{\sigma_t - (1 + \mu)\,p}} - 1\right]$$

The value of σ_t may be taken as 0.8 times the yield point stress (σ_y).

3. *Clavarino's equation.* This equation is also based on the maximum-strain theory of failure, but it is applied to closed-end cylinders (or cylinders fitted with heads) made of ductile material. According to this equation, the thickness of a cylinder,

Oil is frequently transported by ships called tankers. The larger tankers, such as this Acrco Alaska oil transporter, are known as supertankers. They can be hundreds of metres long.

$$t = r_i \left[\sqrt{\frac{\sigma_t + (1 - 2\mu)\,p}{\sigma_t - (1 + \mu)\,p}} - 1\right]$$

In this case also, the value of σ_t may be taken as 0.8 σ_y.

4. *Barlow's equation.* This equation is generally used for high pressure oil and gas pipes. According to this equation, the thickness of a cylinder,

$$t = p.r_o / \sigma_t$$

For ductile materials, $\sigma_t = 0.8\,\sigma_y$ and for brittle materials $\sigma_t = 0.125\,\sigma_u$, where σ_u is the ultimate stress.

Example 7.7. *A cast iron cylinder of internal diameter 200 mm and thickness 50 mm is subjected to a pressure of 5 N/mm². Calculate the tangential and radial stresses at the inner, middle (radius = 125 mm) and outer surfaces.*

Solution. Given : d_i = 200 mm or r_i = 100 mm ; t = 50 mm ; p = 5 N/mm²

We know that outer radius of the cylinder,

$$r_o = r_i + t = 100 + 50 = 150 \text{ mm}$$

Tangential stresses at the inner, middle and outer surfaces

We know that the tangential stress at any radius x,

$$\sigma_t = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right]$$

$\therefore$ Tangential stress at the inner surface (*i.e.* when $x = r_i = 100$ mm),

$$\sigma_{t(inner)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} = \frac{5\,[(150)^2 + (100)^2]}{(150)^2 - (100)^2} = 13 \text{ N/mm}^2 = 13 \text{ MPa } \textbf{Ans.}$$

Tangential stress at the middle surface (*i.e.* when $x = 125$ mm),

$$\sigma_{t(middle)} = \frac{5\,(100)^2}{(150)^2 - (100)^2}\left[1 + \frac{(150)^2}{(125)^2}\right] = 9.76 \text{ N/mm}^2 = 9.76 \text{ MPa } \textbf{Ans.}$$

and tangential stress at the outer surface (*i.e.* when $x = r_o = 150$ mm),

$$\sigma_{t(outer)} = \frac{2\,p\,(r_i)^2}{(r_o)^2 - (r_i)^2} = \frac{2 \times 5\,(100)^2}{(150)^2 - (100)^2} = 8 \text{ N/mm}^2 = 8 \text{ MPa } \textbf{Ans.}$$

Radial stresses at the inner, middle and outer surfaces

We know that the radial stress at any radius x,

$$\sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right]$$

$\therefore$ Radial stress at the inner surface (*i.e.* when $x = r_i = 100$ mm),

$$\sigma_{r(inner)} = -p = -5 \text{ N/mm}^2 = 5 \text{ MPa (compressive) } \textbf{Ans.}$$

Radial stress at the middle surface (*i.e.* when $x = 125$ mm)

$$\sigma_{r(middle)} = \frac{5\,(100)^2}{(150)^2 - (100)^2}\left[1 - \frac{(150)^2}{(125)^2}\right] = -1.76 \text{ N/mm}^2 = -1.76 \text{ MPa}$$

$$= 1.76 \text{ MPa (compressive) } \textbf{Ans.}$$

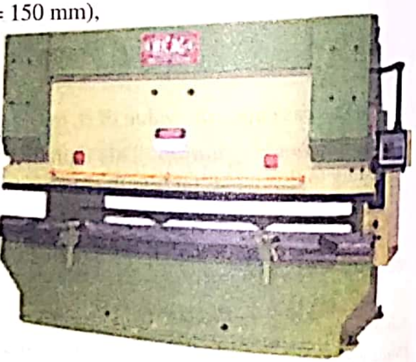
and radial stress at the outer surface (*i.e.* when $x = r_o = 150$ mm),

$$\sigma_{r(outer)} = 0 \textbf{ Ans.}$$

Example 7.8. *A hydraulic press has a maximum capacity of 1000 kN. The piston diameter is 250 mm. Calculate the wall thickness if the cylinder is made of material for which the permissible strength may be taken as 80 MPa. This material may be assumed as a brittle material.*

Solution. Given : $W = 1000$ kN $= 1000 \times 10^3$ N ; $d = 250$ mm ; $\sigma_t = 80$ MPa $= 80$ N/mm²

First of all, let us find the pressure inside the cylinder (p). We know that load on the hydraulic press (W),



Hydraulic Press

$$1000 \times 10^3 = \frac{\pi}{4} \times d^2 \times p = \frac{\pi}{4}\,(250)^2\,p = 49.1 \times 10^3 p$$

$\therefore$ $p = 1000 \times 10^3 / 49.1 \times 10^3 = 20.37$ N/mm²

Let r_i = Inside radius of the cylinder $= d/2 = 125$ mm

We know that wall thickness of the cylinder,

$$t = r_i\left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] = 125\left[\sqrt{\frac{80 + 20.37}{80 - 20.37}} - 1\right] \text{ mm}$$

$$= 125\,(1.297 - 1) = 37 \text{ mm } \textbf{Ans.}$$

Example 7.9. *A closed-ended cast iron cylinder of 200 mm inside diameter is to carry an internal pressure of 10 N/mm² with a permissible stress of 18 MPa. Determine the wall thickness by means of Lame's and the maximum shear stress equations. What result would you use? Give reason for your conclusion.*

Solution. Given : $d_i = 200$ mm or $r_i = 100$ mm ; $p = 10$ N/mm² ; $\sigma_t = 18$ MPa $= 18$ N/mm²

According to Lame's equation, wall thickness of a cylinder,

$$t = r_i\left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] = 100\left[\sqrt{\frac{80 + 10}{80 - 10}} - 1\right] = 87 \text{ mm}$$

According to maximum shear stress equation, wall thickness of a cylinder,

$$t = r_i\left[\sqrt{\frac{\tau}{\tau - p}} - 1\right]$$

We have discussed in Art. 7.9 [equation (*iv*)], that the shear stress (τ) is usually taken one-half the tensile stress (σ_t). In the present case, $\tau = \sigma_t / 2 = 18/2 = 9$ N/mm². Since τ is less than the internal pressure ($p = 10$ N/mm²), therefore the expression under the square root will be negative. Thus no thickness can prevent failure of the cylinder. Hence it is impossible to design a cylinder to withstand fluid pressure greater than the allowable working stress for the given material. This difficulty is overcome by using compound cylinders as discussed in Art. 7.10.

Thus, we shall use a cylinder of wall thickness, $t = 87$ mm **Ans.**

Example 7.10. *The cylinder of a portable hydraulic riveter is 220 mm in diameter. The pressure of the fluid is 14 N/mm² by gauge. Determine suitable thickness of the cylinder wall assuming that the maximum permissible tensile stress is not to exceed 105 MPa.*

Solution. Given : $d_i = 220$ mm or $r_i = 110$ mm ; $p = 14$ N/mm² ; $\sigma_t = 105$ MPa $= 105$ N/mm²

Since the pressure of the fluid is high, therefore thick cylinder equation is used.

Assuming the material of the cylinder as steel, the thickness of the cylinder wall (t) may be obtained by using Birnie's equation. We know that

$$t = r_i\left[\sqrt{\frac{\sigma_t + (1 - \mu)\,p}{\sigma_t - (1 + \mu)\,p}} - 1\right]$$

$$= 110\left[\sqrt{\frac{105 + (1 - 0.3)\,14}{105 - (1 + 0.3)\,14}} - 1\right] = 16.5 \text{ mm } \textbf{Ans.}$$

...(Taking Poisson's ratio for steel, $\mu = 0.3$)

Example 7.11. *The hydraulic cylinder 400 mm bore operates at a maximum pressure of 5 N/mm². The piston rod is connected to the load and the cylinder to the frame through hinged joints. Design: 1. cylinder, 2. piston rod, 3. hinge pin, and 4. flat end cover.*

The allowable tensile stress for cast steel cylinder and end cover is 80 MPa and for piston rod is 60 MPa.

Draw the hydraulic cylinder with piston, piston rod, end cover and O-ring.

Solution. Given : $d_i = 400$ mm or $r_i = 200$ mm ; $p = 5$ N/mm² ; $\sigma_t = 80$ MPa $= 80$ N/mm² ; $\sigma_{tp} = 60$ MPa $= 60$ N/mm²

1. *Design of cylinder*

Let d_o = Outer diameter of the cylinder.

We know that thickness of cylinder,

$$t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] = 200\left[\sqrt{\frac{80+5}{80-5}} - 1\right] \text{ mm}$$

$$= 200\,(1.06 - 1) = 12 \text{ mm } \textbf{Ans.}$$

∴ Outer diameter of the cylinder,

$$d_o = d_i + 2t = 400 + 2 \times 12 = 424 \text{ mm } \textbf{Ans.}$$

2. *Design of piston rod*

Let d_p = Diameter of the piston rod.

We know that the force acting on the piston rod,

$$F = \frac{\pi}{4}(d_i)^2 p = \frac{\pi}{4}(400)^2\, 5 = 628\,400 \text{ N} \quad ...(i)$$

We also know that the force acting on the piston rod,

$$F = \frac{\pi}{4}(d_p)^2 \sigma_{tp} = \frac{\pi}{4}(d_p)^2\, 60 = 47.13\,(d_p)^2 \text{ N} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$(d_p)^2 = 628\,400/47.13 = 13\,333.33 \quad \text{or} \quad d_p = 115.5 \text{ say } 116 \text{ mm } \textbf{Ans.}$$

3. *Design of the hinge pin*

Let d_h = Diameter of the hinge pin of the piston rod.

Since the load on the pin is equal to the force acting on the piston rod, and the hinge pin is in double shear, therefore

$$F = 2 \times \frac{\pi}{4}(d_h)^2 \tau$$

$$628\,400 = 2 \times \frac{\pi}{4}(d_h)^2\, 45 = 70.7\,(d_h)^2 \quad ...(\text{Taking } \tau = 45 \text{ N/mm}^2)$$

∴ $$(d_h)^2 = 628\,400/70.7 = 8888.3 \quad \text{or} \quad d_h = 94.3 \text{ say } 95 \text{ mm } \textbf{Ans.}$$

When the cover is hinged to the cylinder, we can use two hinge pins only diametrically opposite to each other. Thus the diameter of the hinge pins for cover,

$$d_{hc} = \frac{d_h}{2} = \frac{95}{2} = 47.5 \text{ mm } \textbf{Ans.}$$

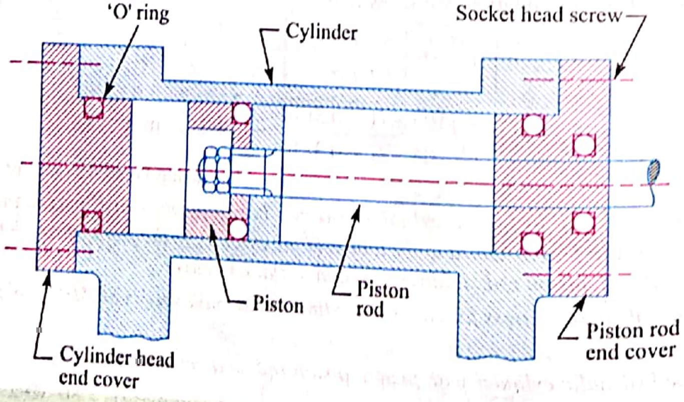


Fig. 7.7

4. *Design of the flat end cover*

Let t_c = Thickness of the end cover.

We know that force on the end cover,

$$F = d_i \times t_c \times \sigma_t$$

$$628\,400 = 400 \times t_c \times 80 = 32 \times 10^3\, t_c$$

∴ $$t_c = 628\,400 / 32 \times 10^3 = 19.64 \text{ say } 20 \text{ mm } \textbf{Ans.}$$

The hydraulic cylinder with piston, piston rod, end cover and *O*-ring is shown in Fig. 7.7.

7.10 Compound Cylindrical Shells

According to Lame's equation, the thickness of a cylindrical shell is given by

$$t = r_i\left(\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right)$$

From this equation, we see that if the internal pressure (p) acting on the shell is equal to or greater than the allowable working stress (σ_t) for the material of the shell, then no thickness of the shell will prevent failure. Thus it is impossible to design a cylinder to withstand internal pressure equal to or greater than the allowable working stress.

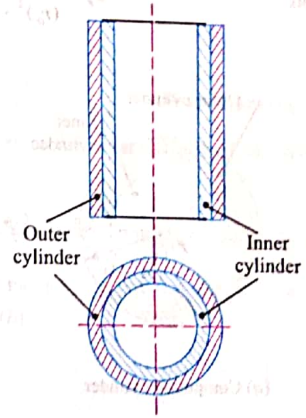


Fig. 7.8. Compound cylindrical shell.

This difficulty is overcome by inducing an initial compressive stress on the wall of the cylindrical shell. This may be done by the following two methods:

1. By using compound cylindrical shells, and
2. By using the theory of plasticity.

In a compound cylindrical shell, as shown in Fig. 7.8, the outer cylinder (having inside diameter smaller than the outside diameter of the inner cylinder) is shrunk fit over the inner cylinder by heating and cooling. On cooling, the contact pressure is developed at the junction of the two cylinders, which induces compressive tangential stress in the material of the inner cylinder and tensile tangential stress in the material of the outer cylinder. When the cylinder is loaded, the compressive stresses are first relieved and then tensile stresses are induced. Thus, a compound cylinder is effective in resisting higher internal pressure than a single cylinder with the same overall dimensions. The principle of compound cylinder is used in the design of gun tubes.

In the theory of plasticity, a temporary high internal pressure is applied till the plastic stage is reached near the inside of the cylinder wall. This results in a residual compressive stress upon the removal of the internal pressure, thereby making the cylinder more effective to withstand a higher internal pressure.

7.11 Stresses in Compound Cylindrical Shells

Fig. 7.9 (*a*) shows a compound cylindrical shell assembled with a shrink fit. in the previous article that when the outer cylinder is shrunk fit over the innr pressure (p) is developed at junction of the two cylinders (*i.e.* at radius r_2) a and (*c*). The stresses resulting from this pressure may be easily determined b

According to this equation (See Art. 7.9), the tangential stress at ar

$$\sigma_t = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} + \frac{(r_i)^2 (r_o)^2}{x^2}$$

...ives by